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Seismic properties of solar-type stars

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Abstract. The forthcoming space observations of stellar variability (MOST, COROT and EDDINGTON) will soon provide accurate seismic data on solar-type stars. In this contribution we present a preliminary study on asteroseismic inversion techniques based on the analysis of the well known seismic parameters: large ($\Delta\nu_{n,l}$) and small ($\delta\nu_{n,l}$) frequency separations. For a set of models describing main-sequence low- and moderate-mass stars, we have calculated the so-called *asteroseismic HR diagrams*, underlining the role of the l-dependent part of the averaged small separation as a probe of the inner regions of the star. We have also shown how the properties of helium ionization regions can be determined from the characteristics of the oscillatory components in $\Delta\nu(\nu)$.

Key words. asteroseismology – solar-like oscillations

1. Introduction

The structure of an oscillation spectrum of any star exhibiting solar-like oscillations can be simply characterized by the large ($\Delta\nu_{nl}$) and the small ($\delta\nu_{nl}$) frequency separations, defined as:

$$\Delta\nu_{n,l} \equiv \nu_{n,l} - \nu_{n-1,l} \tag{1}$$

$$\delta\nu_{n,l} \equiv \nu_{n,l} - \nu_{n-1,l+2} \tag{2}$$

where ν is the frequency of the mode, identified by the radial order n and degree l.
The frequency separations are indicators of the seismic properties of stars, that is, they are probes of stellar interiors, supplying us with information on the internal constitution of stars which cannot be obtained by other means. The forthcoming ultra-high precision relative photometric observations from space (MOST, COROT and EDDINGTON) will allow the detection of the required low degree solar-like oscillations (relative amplitude $\sim 10^{-6}$) in main sequence stars of low- and moderate-mass.

2. Asteroseismic HR diagrams

Tassoul (1980) obtained, by an asymptotic approximation valid for high-order pressure modes, the dependence of the large and small separations on sound speed:

$$\Delta\nu \simeq \left(2\int_0^R \frac{\mathrm{d}r}{c}\right)^{-1} \tag{3}$$

$$\delta\nu_{nl} \simeq -(4l+6)\frac{\Delta\nu}{4\pi^2\nu_{nl}}\int_0^R \frac{\mathrm{d}c}{\mathrm{d}r}\frac{\mathrm{d}r}{r} \tag{4}$$

where c is the adiabatic sound speed.

The large separation is then a measure of sound travel time between the surface of the star and the center; whereas $\delta\nu_{nl}$ is sensitive to

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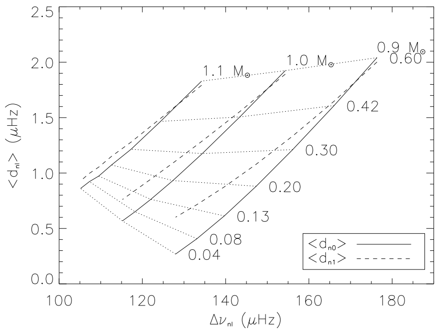

Fig. 1. d_{nl} is calculated both with modes of degree $l = 0, 2$ ($< d_{n0} >$, continuous lines) and $l = 1, 3$ ($< d_{n1} >$, dashed lines). Dotted lines represent constant central hydrogen abundance (*isopleths*).

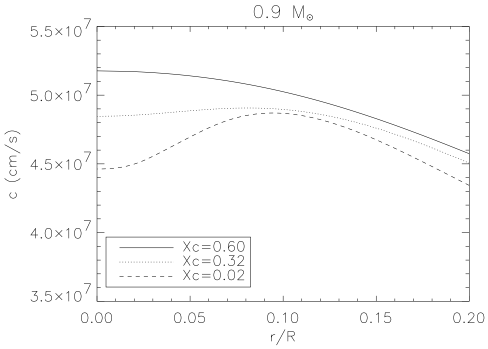

Fig. 2. Sound speed near the center of a 1 $M_\odot$ star during the main sequence. The increase of the mean molecular weight during hydrogen burning is responsible for the decrease of the sound speed towards the center.

the structure of the stellar core. We define the small separation as follows:

$$d_{nl} = \frac{1}{2l+3}(\nu_{n,l} - \nu_{n-1,l+2}), \qquad (5)$$

which is expected to be l-independent, according to the approximated expression of Eq. (4). The combined plot of averaged values of $\Delta\nu$ and d_{nl} was first suggested by Christensen-Dalsgaard (1988) as a possible means to infer the mass and age of a star from asteroseismic data.

We have considered a set of main-sequence stars with masses in a range between 0.9 and 2.0 $M_\odot$ with initial hydrogen abundance X=0.70, metallicity Z=0.02 and mixing length parameter $\alpha = 1.83$. All models and oscillation spectra have been calculated using Christensen-Dalsgaard (1982) stellar structure and evolution code and Christensen-Dalsgaard stellar adiabatic oscillations code.In Fig. 1 we present an example of our calculations, that is, the evolutionary tracks for 0.9, 1.0 and 1.1 $M_\odot$ main-sequence stars.

For each model considered we have calculated its acoustic spectrum and reported on the x- and y-axis $\Delta\nu_{nl}$ and d_{nl}, respectively, averaged on high order modes ($15 \leq n \leq 25$).

We find that, as shown in Fig. 1, $< d_{n0} >$ generally differs from $< d_{n1} >$. The difference $< d_{n0} > - < d_{n1} >$ could be qualitatively explained by taking into account the fact that modes of different degree sound differently and at different depths the internal structure of the star, particularly the region of a star close to the center, where the sound speed decreases during evolution (see captions to Fig. 2 and 3). We have found not sufficient to restrict the domain of the integral in Eq. (4) to the l-dependent region where the mode is trapped, according to an approximation of the small separation given by Gough (2003). In fact, both approximations considered so far suffer from neglecting the effects of the perturbations in the gravitational potential, which give a substantial contribution to the small separation. We believe that a different theoretical approach (Roxburgh & Vorontsov 1994), could provide the required relation between $< d_{n0} > - < d_{n1} >$ and the characteristics of the stellar core.

3. Signatures of helium ionization zones in $\Delta\nu$

Localized variations in the structure of stars, such as the one that occurs in the region of the second ionization of helium, create a peculiar signal in the frequencies of oscillation. The properties of such a signal are related to the location and thermodynamic properties of the star at the layer where the sharp variation occurs.

Using a variational principle (Monteiro &

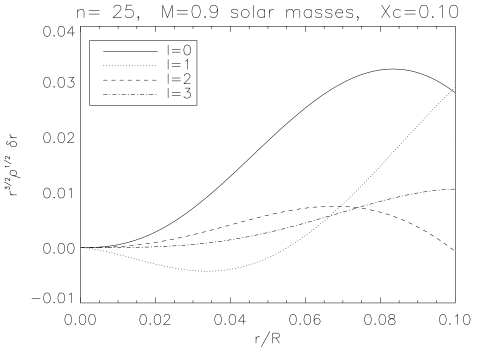

Fig. 3. Behaviour of the eigenfunctions associated with modes used to calculate $d_{\bar{n}0}$ and $d_{\bar{n}1}$, identified by the integers $(n, l) = (\bar{n}, 0), (\bar{n}-1, 2)$ and $(\bar{n}, 1), (\bar{n}-1, 3)$; here $\bar{n} = 25$. (see Eq. (5)). In the region where sound speed rapidly changes (see Fig. 2) modes of different l have significatively different amplitudes.

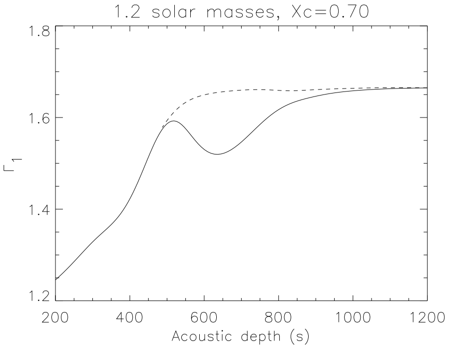

Fig. 4. The behaviour of Γ_1 in the "real" model (continuous line) and in the fictitious smooth model (dashed line). The local minimum at $\simeq 650$ s is related to the second helium ionization zone.

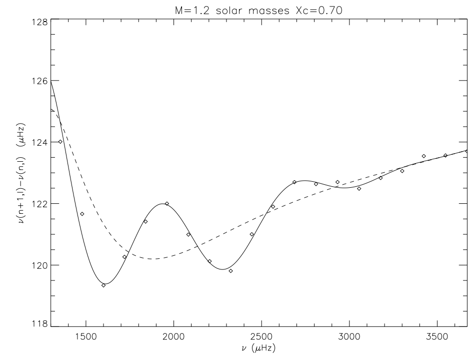

Fig. 5. The combination (continuous line) of a smooth signal (dashed line) and the oscillatory component in Eq. (6) reproduces the behaviour of $\Delta\nu(\nu)$ (circles) in a 1.2 $M_\odot$ ZAMS star.

Thompson 1998) it is possible to determine the relation between the characteristics of the ionization zone, considered as a localized perturbation on an otherwise smooth structure (see Fig. 4), and the signal appearing in both the frequencies and $\Delta\nu$.

Considering the case of the "bump" in Γ_1 related to the second helium ionization, an explicit expression for the oscillatory signal, derived in the asymptotic approximation, for low-degree p modes is given by (Monteiro & Thompson 1998):

$$\delta\omega_{\rm p} \sim a_{\rm d}\frac{\sin^2(\beta\omega)}{\beta\omega}\cos\left[2(\omega\tau_{\rm d}+\phi_0)\right]\,, \qquad (6)$$

where $\tau_{\rm d}$ is approximately the acoustic depth of the ionization zone, and β and $a_{\rm d}$ are related to the width and depth of the variation in Γ_1.

We have used Eq. (6) as the basis for fitting the oscillatory component in the large frequency separation computed from p-mode spectra: in this way we determined the acoustic depth of the second helium ionization zone (see Fig. 5) for models with $M \leq 1.5\ M_\odot$.

A more general analysis of the effect of the bumps in Γ_1 can be obtained by noting that small variations in the equilibrium structure of the star lead to changes in the frequencies of the form (Gough & Thompson 1991):

$$\frac{\delta\omega_{nl}}{\omega_{nl}} = \int_0^R \left[A\frac{\delta_r\Gamma_1}{\Gamma_1}(r) + B\frac{\delta_r u}{u}(r)\right]{\rm d}r\ . \qquad (7)$$

Here $A = K^{nl}_{\Gamma_1,u}(r)$, $B = K^{nl}_{u,\Gamma_1}(r)$, $u = p/\rho$, where p and ρ are pressure and density, and the kernels $K^{nl}_{\Gamma_1,u}$ and K^{nl}_{u,Γ_1} are computed from the reference model; δ_r denotes model differences at fixed fractional radius. In the case considered, the variations in the equilibrium model are sought relative to a fictitious smooth model with suppressed helium ionization, that is, $\delta_r\Gamma_1/\Gamma_1$ in Eq. (7) represents $(\Gamma_{1,smooth}(r) -$

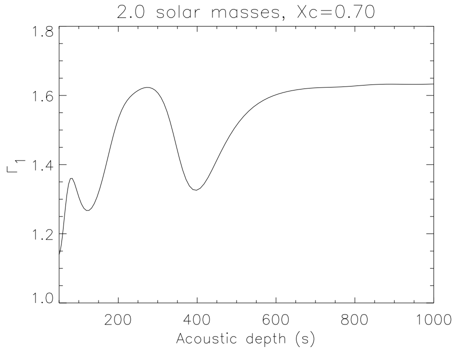

Fig. 6. The behaviour of Γ_1 in a model for a 2.0 $M_\odot$ ZAMS stars, the minimum at an acoustic depth $\simeq 130$ s is related to the region of first helium ionization.

$\Gamma_1(r))/\Gamma_1(r)$ (see Fig. 4). In models of mass $M > 1.5$ $M_\odot$, $\delta\omega_{nl}$ obtained in Eq. (7) allowed us (Miglio et al. 2003) to relate a different behaviour of the oscillatory component in $\Delta\nu$ to another sharp change in Γ_1, namely where the first ionization of helium is located (see Fig. 6).

4. Conclusions

We have presented some of the techniques that will help to develop asteroseismology of solar-type stars from space, that is, to reveal, with high resolution, the properties of stellar interiors. As suggested at this conference, observed low-degree solar oscillation frequencies represent a first valuable data set to test the effectiveness of these techniques. A study on this topic is in progress.

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