A new Period-Radius relation for Galactic Classical Cepheids

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Abstract. We discuss a new Period-Radius (PR) relation for Galactic Classical Cepheids, obtained by means of a new version of the CORS method which has been modified in order to be run with the Strömgren photometric system. The major change consists in the calibration of the Surface Brightness as a function of the two “reddening free” colour indexes $[c1]$ and $[m1]$, by means of the model atmospheres by Castelli et al. (1997).
In this contribution we first briefly discuss some numerical experiments performed on the basis of synthetic Cepheid light curves to test the accuracy of the method, and then report the Period-Radius relation for Classical Cepheids obtained by applying the new method to a sample of Galactic Cepheids.

Key words. stars: distance – stars: fundamental parameters – stars: variables: Cepheids

1. Introduction

As well known, Cepheid variables are the most important distance indicators within the Local Group and (thanks to space observations) beyond. Therefore, an accurate knowledge of their properties is of crucial importance. In this context, the determination of Cepheid radii and the consequent derivation of a period-radius relation (PR), are relevant to determine the distances, the masses and other physical parameters of these variables.

To determine Cepheid radii, we have adopted a modification of the Baade-Wesselink method called CORS (see Caccin et al., 1981). In particular, the availability of a new set of homogeneous data in Strömgren photometric system for a relevant sample of Galactic Cepheids (Arellano Ferro et al. 1998), and the possibility to define the two reddening free colour indexes $[m1]$ and $[c1]$ (Crawford & Mandwewala, 1976) which, for Cepheids, are sensitive to the effective temperature and gravity respectively, suggested us to modify the original CORS method in order to make it work with the Strömgren photometric system.

2. The modified CORS method and tests with synthetic Cepheids

From the definition of the CORS Method (see Caccin et al. 1981 for details):

$$ q \int_{0}^{1} \ln \left( R_0 - k P \int_{\phi_0}^{\phi} v(\phi') d\phi' \right) C_i d\phi $$
Fig. 1. Light, colour and radial velocity curves for the model at 6.25$M_\odot$ and $T_{\text{eff}}=5800K$. The left panel shows the synthetic curves; the central and right panels show the same curves added with increasing gaussian noise in order to simulate good and fair quality data respectively.

\[-B + \Delta B = 0\]  \hspace{1cm} (1)

it follows that the key point is the determination of the so called $\Delta B$ term, which is defined as $\Delta B = \int_0^1 C_{ij}(\phi) S'_V(\phi) d\phi$, where $C_{ij}$ is a color and $S_V$ is the Surface Brightness. To this aim, the first step is the calibration of the Surface Brightness as function of two properly chosen colour indexes (see Onnembo et al. 1985) which, in our case, are $[m1]$ and $[c1]$. We have achieved this by fitting a $4^{th}$ degree polynomial (least square fit) to the grids of model atmospheres by Castelli et al. (1997). Once calibrated the Surface Brightness and determined in turn the the $\Delta B$ term, the modified CORS method is ready to be applied.

3. Tests of the method by means of synthetic Cepheids

Before using the new method with actual data, we have verified its accuracy by applying it to the synthetic light, color and radial velocity curves predicted by Cepheid full amplitude, nonlinear, convective pulsation models (see figure Fig. 1 left panel for an example, and Bono, Marconi, Stellingwerf, 1999; Bono, Castellani, Marconi 2002 for details on the models).
In particular we have selected ten models at solar chemical composition ($Y = 0.28$, $Y = 0.02$) and stellar masses ranging from 5 to 9 $M_{\odot}$. At fixed mass we generally selected three models with different temperatures in order to span the width of the predicted instability strip. The period of selected models roughly ranges from 3.5 to 62 days. Details on the tests will be given elsewhere (Ruoppo et al. 2003, in preparation, R03 hereafter). Here we only summarize the following relevant points:

- in order to verify the consistency of the new method, we applied it directly to the synthetic curves produced by pulsation models. As a result we find that we can reproduce the theoretical radii within 1%.
- in order to estimate to which extent the method is sensitive to random errors in the observations, we reduced the number of phase points to 30-35, (extracting randomly the phases) and added gaussian noise to simulate good and fair quality data (see Fig. 1 center and right panels respectively for an example. Details in R03). As a result, the average error on the radius is 4.2 % and 9 % for good and fair quality data respectively, thus pointing out that too scattered observed data have to be discarded.

4. The Period-Radius relation for Galactic Cepheids

Once the new CORS method has been modified, we have applied it to a sample of 52 Galactic Cepheids for which we could find in the literature reasonable Strömgren photometry and radial velocity curves (details on the sources in R03).

The obtained radii are reported in Fig. 2 as a function of the pulsation period. The solid lines superimposed on the data represent a least square fit which leads to the following Period-Radius relation:

$$\log R = \left(0.69 \pm 0.09\right) \log P + 1.22 \pm 0.08$$

We notice that this relation was obtained excluding the following stars:

- DT Cyg, FF Aql, BB Gru and V440 Per, because they are first overtone pulsators;
- TU Cas, V367 Sct and BQ Ser, because they are double mode Cepheids;
- U Aql, GI Car, SU Cyg, VZ Cyg, ζ Gem, T Mon, Y Sct, U Sgr and W Sgr, because they are members of binary systems.

We note that for other 11 stars our program failed to reach the convergence probably because of the poor quality of the
Table 1. Comparison of Period-Radius coefficients (log $R = a \log P + b$) between this paper and selected works in literature.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>Source</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.751</td>
<td>± 0.026</td>
<td>1.070 ± 0.008</td>
<td>LS</td>
</tr>
<tr>
<td>0.750</td>
<td>± 0.024</td>
<td>1.075 ± 0.007</td>
<td>GFG</td>
</tr>
<tr>
<td>0.606</td>
<td>± 0.037</td>
<td>1.263 ± 0.033</td>
<td>RBMR</td>
</tr>
<tr>
<td>0.659</td>
<td>± 0.051</td>
<td>1.148 ± 0.042</td>
<td>AFR</td>
</tr>
<tr>
<td>0.655</td>
<td>± 0.006</td>
<td>1.188 ± 0.008</td>
<td>BCM</td>
</tr>
<tr>
<td>0.69</td>
<td>± 0.09</td>
<td>1.12 ± 0.08</td>
<td>This Work new CORS (with $\Delta B$)</td>
</tr>
</tbody>
</table>

Source: Arellano Ferro & Rosenzweig 2000 (AFR); Laney & Stobie 1995 (LS); Gieren et al. 1998 (GFG); Ripepi et al. 1997 (RBMR); Bono et al. 1998 (BMC).

Unfortunately, the remaining sample of Cepheids does not include long period objects (see R03 for details). In any case, our Period-Radius relation, when compared with literature results (cf. Table 1) is in agreement, within the errors with previous works. However we note that the errors of our relation are larger. This is probably due to the relatively less extensive sample of investigated Cepheids and to the lack of long period objects. In the future we shall attempt to address these points in order to improve our PR relation and provide a calibration of the Period-Luminosity relation.

References