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Collisional filtration model for electrons in the Solar Transition Region

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Abstract. In this work we have looked for a solution of the Boltzmann equation in the solar transition region, trying to solve it with a Monte Carlo method. As a solution, we have obtained a temperature enhancement, a density drop and an electron heat flux towards the innersphere, showing a departure from the classical transport theory.

1. Introduction

A challenging problem in solar physics is represented by the attempt to understand the reasons for the steep temperature increase and density drop over a very short (with respect to solar scale lengths) distance (some thousand kilometers) in the transition region (TR). The electron mean free path increases from some hundred centimeters in chromosphere to several thousand kilometers in corona (Mariska 1992), realizing a matching between a collisional medium, where a fluid description is adequate, and a medium where collisionless transport processes are expected to be dominant. For the above mentioned reasons, two opposite approaches to this problem have been pursued: the classical hydrodynamic approach (Spitzer&Ḧarm 1953; Braginskii 1965) yielding to the fluid models and the collisionless one yielding to the exospheric models (Maksimovic et al. 1997). Collisional fluid descriptions for modelling the solar transition region are limited because of the presence of huge temperature and density gradients and because the thermal Knudsen numbers K_T are large enough for the classical transport coefficients to become modified (Gray&Kilkenny 1980). In the collisionless approach the Boltzmann equation for the particle flow is reduced to the Vlasov equation and any function of the total energy is a solution; e.g., starting with a Maxwellian distribution function (DF) at the lower boundary, at each height a Maxwellian is found with the same temperature. To overcome the difficulty to reproduce the temperature jump, in the collisionless approach, Scudder (Scudder 1992a,b) proposed a new mechanism called *the velocity filtration effect*: when using a generalized Lorentian (or Kappa) distribution for particles at the lower boundary, this excess is recovered in the higher regions, giving rise to a temperature enhancement. Collisionless exospheric models are limeted because collisions are not negligible and because they require non-thermal distribution functions (like Kappa-functions) to reproduce a temperature gradient. For the above mentioned limitations, a complete description of the solar TR requires a kinetic model taking collisions into account; so, it is natural to consider the Boltzmann equation. Due to complexity of the physical problem and of the mentioned mathematical equation, we have made some simplifying assumptions, described in the next section.

2. Numerical simulation

Let us assume a stationary case, a planar one-dimensional geometry in the physical space and a cylindrical one around the vertical direction in the velocity space. In this case the DF depends on three variables, $f(x, u_\parallel, u_\perp)$, where x is the vertical coordinate (the distance from the solar surface), $u_\parallel = u_x$ is the velocity parallel to the vertical direction and $u_\perp$ is the modulus of the velocity perpendicular to this direction; then, the stationary Boltzmann equation for the electrons is written $u_x \frac{df(x,\mathbf{u})}{dx} + \frac{eE}{m_e}\frac{df(x,\mathbf{u})}{du_x} = \int d\mathbf{v} \int_0^{b_{max}} bdb \int_0^{2\pi} |\mathbf{u} - \mathbf{v}| d\epsilon (f(x,\mathbf{u}')f(x,\mathbf{v}') - f(x,\mathbf{u})f(x,\mathbf{v}))$, where e and m_e are the electron charge and mass, respectively. The right hand-side term is the collisional integral, in which two populations of particles are considered: the target ones with velocity $\mathbf{v}$ and the incident ones with velocity $\mathbf{u}$; $\mathbf{v}'$ and $\mathbf{u}'$ are their velocities after a collision, respectively; b is the impact parameter and, since we describe Coulomb collisions between electrons, the maximum impact parameter b_{max} is the Debye radius; ϵ is the angle made by the collision plane and an arbitrary plane, measured in a plane normal to the relative velocity vector $|\mathbf{u} - \mathbf{v}|$ (see figure 2 of (Haviland 1965)). To avoid solving the Boltzmann equation for the protons, we have made the hypotheses of charge neutrality, equal bulk velocity V and temperature T in the momentum equations of the two kinds of particles; from these, we can obtain the expression of the electric field, where the electrons move $E = \frac{m_i - m_e}{2e}(V\frac{dV}{dx} + \frac{GM_S}{x^2})$, obtained in 1972 by Lemaire and Scherer. We studied the solution of the above Boltzmann equation in a region which starts at 2000 Km above the photosphere and extends up to about 7×10^4 Km in the corona. At the lower boundary we supposed that distribution function for the incoming electrons is Maxwellian; at the upper boundary, we assumed that the motion outside the region under study is ballistic, so that electrons which arrive with a velocity less than the escape one, come back to this boundary with a reversed sign of velocity (the escape velocity is considered as a parameter of the problem). Solving the above Boltzmann equation with the above reported boundary conditions, we obtain electron distribution function at each level of the simulation box, which in turn allows us to determine the density, temperature, bulk velocity and heat flux profiles.

To solve the Boltzmann equation we use a Montecarlo method widely described in (Haviland 1965); this tecnique shows the distinct advantage that is not necessary to introduce simplifying assumption to evaluate the collisional integral. This method is based on five main points:
(i) about 10^4 electrons are injected in the simulation box at the lower boundary with the assumed DF for positive parallel velocities; (ii) electrons propagate in the simulation box under the effect of the above electric field and are subjected to Coulomb collisions whose rate is determined by the DF of the target electrons; (iii) when particles arrive at the upper boundary we reverse their radial velocity if it is lower than the escape velocity; (iv) we stop to follow the particles trajectory when they arrive either at the lower boundary or to the upper one, but with a radial velocity higher than the escape one; (v) to compute the DF we have divided the phase space in cells and the DF for the incident electrons is computed by accumulating the time spent by the particles in each cell of the phase space.

Clearly, the knowledge of the target DF is needed, so we use an iterative procedure, in which the DF for the incident electrons obtained in the previous iteration is used as the DF of the target electrons in the following iteration. When the distribution functions of two following iterations are almost the same, the convergence has been obtained and we have a solution to the Boltzmann equation. At the first iteration step, we started with a Maxwellian guess target DF and we have used guess density and temperature profiles obtained from the Fourier's law with constant heat flux q_o (Owocki&Canfield 1986); from the guess assumption of constant pressure p_o, the density profile is given by $n(x) = p_o/k_B T(x)$. T_o, q_o and p_o are guess parameters, referred to their value at the lower boundary of the simulation box.

3. Numerical results

Each length in the code is normalized to the Debye radius at the lower boundary r_{Do}, velocities to the elctron thermal velocity at the lower boundary v_{tho} and times to the inverse of the electron plasma frequency at the lower boundary $\omega_{peo} = v_{tho}/r_{Do}$. With this choice, density profiles are normalized to $n^* = r_{Do}^{-3}$, temperatures to $T^* = m_e v_{tho}^2/2K_b$ (T^* and T_o have the same value) and the heat fluxes to $q^* = n^* m_e v_{tho}^3$.
The figure 1 shows an example of above mentioned iterative process: 6 iterations were needed to reach the convergence of the moments of the distribution function; the plot on the top left and on the top right are the density and the average temperature, respectively and the plots on the bottom left and on the bottom right are the electron bulk velocity and the electron heat flux, respectively. The guess parameters for this simulation are $T_o = 10^4$K, $p_o = 6.9 \times 10^{-3}$ergcm^{-3} ($n_o = 5 \times 10^9$cm^{-3}) and $q_o \sim 5 \times 10^5$erg s^{-1}cm^{-2} (see (Owocki&Canfield 1986)). Besides, the density normalization n^* is equal to 10^6cm^{-3} and heat flux normalization q^* is about 150erg s^{-1}cm^{-2}. From the two plots on the top, we can note that density decreases by a factor of about 100 and temperature increases by the same factor, reaching values of 1 million of degrees in corona. As we can see from the fourth panel of the figure 1, we obtain an heat flux peak value of the order of $2000q^* = 3 \times 10^5$erg/cm^{2}sec. If we compare with the free-streaming value of the heat flux coming from corona with an electron thermal velocity v_{th1}, $q_{fs} = n_1 m_e v_{th1}^3 \sim 7 \times 10^6$erg/cm^{2}sec ($n_1$ is electron density at the upper boundary), we find that the electron heat flux is limeted by the value for streaming particles in a neutral gas (Campbell 1984). Another upper limit for the electron transported heat flux is the non collisional value (with the hypothesis of Maxwellian distribution functions at the boundaries) $q_{NC} = K n_L (T_o T_L^{1/2} - T_L T_o^{1/2}) \sim -4.3 \times 10^5$erg/cm^{2}sec ($K$ is a constant, T_o and T_L are the temperatures at the boundaries of the simulation box and n_L is the density at the upper side (Landi&Pantellini 2001)); the value is greater than the obtained one. So, the general behaviour of the system is not that of a collisionless plasma. Finally, the profile of the classical Spitzer-Härm heat flux (Lie-Svendsen et al. 1999) given by the expression $q_{SH} = -k_e T^{5/2} \frac{dT}{dx}$, (calculated with the obtained temperature and density profiles) is very close to the obtained one in corona, where temperature and density gradients are very small, but near the transition region, where the hypoteses for the classical heat conduction don't hold, the profile shows a significant departure from the obtained one. Figure 1 shows the evolution of the computed electron DF, of the last iteration for the case $n_o = 5 \times 10^9$cm^{-3}, versus parallel velocity normalized to v_{tho}, at four distances from the solar surface integrated on perpendicular velocities: at 7×10^4 Km (the upper boundary) the DF is not symmetric, as we can see from the comparison with a Maxwellian distribution function with the same temperature and density (dotted line). It is probably due to the lost particles having a parallel velocity greater than the escape one. As the altitude decreases

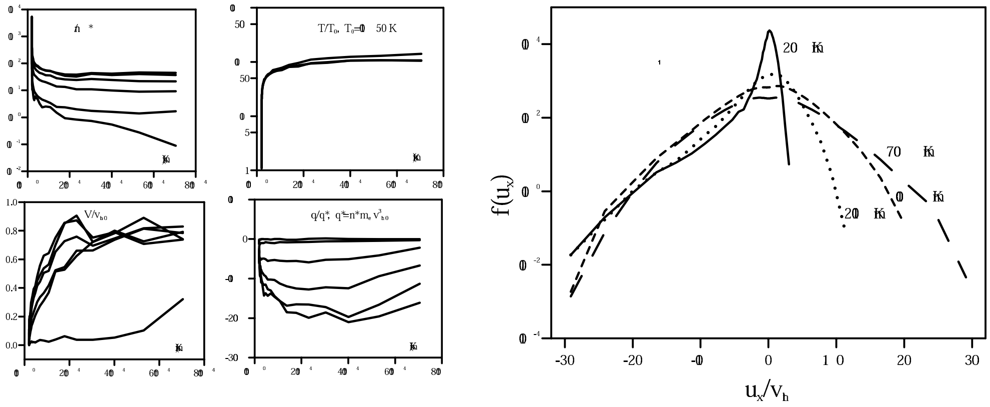

Fig. 1. On the left: iterations for the moments of the distribution function, normalized at their value at the lower boundary of the simulation box. On the right: the distribution function of the last itaration, displayed for 4 distances above the solar surface. The dot line is explained in the text.

(10^4Km, 2.1×10^3Km, 2×10^3Km) the electron DF becomes more non-Maxwellian, in fact it exhibits an anisotropic, high-velocity tail going downward from corona to chromosphere, carring a heat flux, as it is possible to note in the fourth panel of the figure 1. In the same time an enthalpic flux $\frac{3}{2}nVK_bT + pV$ appears in order to balance the heat flux. At the lower boundary (2×10^3Km) the distribution for positive velocities is consistent with the assumed Maxwellian conditions at the injection layer.

4. Discussion and Conclusions

Concerning the broadening of the electron DF as the altitude increases, we think that a filtration process occurs in our system and this effect is mainly carried out by the collisions (besides the attractive potential), therefore in our case we do not need to inject any energy density in the tails at the base of TR: an electron which is going towards the corona has a great probability to collide with a hotter target electron, so it could gain energy and its free path could became very large (the free path $\lambda \sim v^4$). In this way it will suffer few collisions during its path and can have a quasi-ballistic motion up to corona, where the rate of collisions is decreased; so, if this electron has enough energy to overcome the potential well, it arrives to the upper boundary with large velocities. Particles which do not gain energy with collisions display a diffusive motion which keeps them for longer times in the lower regions. As result, in higher regions we mainly find high energy particles which in turn gives rise an enhancement of temperature.

Clearly, we have made several simplyfing assumptions; on of these is to not consider electron-proton collisions; however, even if protons do not matter for the energy distribution, they could be important for the angular distribution and for the isotropy of the electrons (Lie-Svendsen et al. 2000).

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