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Memorie della

Geomagnetic Polarity Reversals and Stochastic Resonance

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Abstract. A special class of noise-induced cooperative transitions is represented by a nonlinear process (the stochastic resonance) in which the coupling between a small periodic undetectable signal and the intrinsic random fluctuations enhances the transition probability in inherent bistable stochastic systems.
Here, analysing the probability distribution function of polarity residence-times, we show that this phenomenon has features that are similar to those of the stochastic resonance. In detail, evidence of stochastic synchronisation is found and its relevance is discussed in connection with changes of Earth's orbital parameters.

Key words. Earth: geodynamo - Earth: paleomagnetic polarity reversals

1. Introduction

One of the most fashion discoveries of paleomagnetic studies is that the Earth's dipolar magnetic field has reversed its polarity many hundreds of times in the past. In the last three decades, many hypothesis were put forward on the possible processes responsible for the reversal of the Earth's magnetic field (Merrill et al. 1996; Seki and Ito 1993; Consolini et al. 2000). Among these, one of the most fascinating and oldest hypothesis involves a possible link among the occurrence of geomagnetic reversals, paleoclimatic changes, and Earth's orbital parameters (Malkus 1968; Doake 1977; Worm 1997; Yamazaki and Oda 2002; Zachos et al. 2001) suggesting that the orbital forcing and/or paleoclimatic changes may energize the geodynamo. Nevertheless, because the evidence of a clear correlation between the Earth's orbital changes and the geomagnetic polarity reversals is extremely hard to find, the general consensus is that this hypothesis has to be untenable. Recently, Yamazaki and Oda (2002), studying a record of the inclination and intensity of the Earth's magnetic field during the past 2.25 million years, found the presence of $\sim$ 100 kyr periodicy in inclination and intensity. This result suggests that geomagnetic field might be, someway, modulated by orbital eccentricity. Thus, the possible link between the geomagnetic polarity reversals and the Earth's orbital parameters is still

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an open problem. Recently, it has been shown that geomagnetic polarity reversal may be approached in terms of the motion of a thermally activated strongly damped particle in a bistable potential well with minima representing normal and reversed polarity (Hoyng et al. 2001). In this framework, reversals are spontaneous, fast, random transitions without an external driving mechanism. According to Kramers reaction rate theory (Hänggi et al. 1990) this stochastic approach to geomagnetic polarity reversals predicts a Poissonian distribution function of polarity intervals. Here, we present a different framework to approach the geomagnetic polarity reversals and the link with the Earth's orbital parameter. We believe that a possible way to reconcile such a stochastic point of view with the existence of a relationship between reversals and Earth's orbital parameter changes is to claim the occurrence of a stochastic resonance process (Wiesenfeld and Moss 1995; Gammaitoni et al. 1998). The phenomenon of stochastic resonance (SR) is a simple and intuitive effect that can occur in bistable systems, in systems in which there is a threshold or in excitable systems.

SR requires three basic ingredients: 1) an energetic activation barrier or, more generally, a form of threshold; 2) a weak coherent input (i.e. a periodic signal); 3) a source of noise that is inherent in the system, or that adds to the coherent input. SR is wery well described by the 1d motion of an overdamped point mass in a double-well potential profile in the presence of a random noise and of a weak periodic force (i.e. a force not able to cause transition when applied alone). In such a picture, the addition of an optimal amount of noise to the system may enhance the synchronous transitions between the two potential minima. In detail, the statistical synchronization takes place when the average waiting time T_m between two interwell transitions is approximatively half the period T_Ω of the periodic forcing. This means that time scale matching condition for the SR is: $2T_m = T_\Omega$. A possible way to reveal the occurrence of stochastic resonance is provided by the study of the distribution function of the time intervals between two successive switches (residence times). As a matter of fact, in the SR case, the residence time distribution function (RTDF) shows a sequence of exponentially decaying peaks which scale with odd numbers according to:

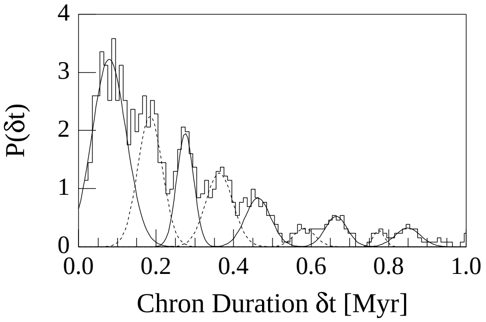

Fig. 1. The normalized RTDF of the polarity time intervals. Solid and dotted lines refer to the individual Gaussian functions.

$$T_j = (2j+1)\frac{T_\Omega}{2} \tag{1}$$

with $j = 0, 1, 2, ..., N$.

2. Data Set and Analysis

To investigate the RTDF in the case of geomagnetic polarity reversals we considered the geomagnetic polarity time scale covering the period from present days back to $\sim$ 166 Myr. The geomagnetic polarity timescale results from the merging of two scales compiled by Cande and Kent (1992, 1995) and by Ogg (1995). Because of the poor statistics to estimate the RTDF of the geomagnetic polarity intervals we used a moving box technique, defining the probability density in each box as follows:

$$p(x_i) = \frac{n_{events}\left|_{x_i-\Delta x/2}^{x_i+\Delta x/2}\right.}{N\Delta x} \tag{2}$$

where $x_i = (0.02 + 0.01i)$ Myr with $i \in I^+$, and $\delta x = 0.04$ Myr. The window Δx was optimised to have a good statistics in each

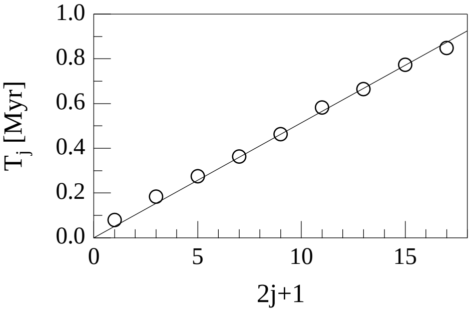

Fig. 2. The peak position of each Gaussian function plotted versus the odd numbers. Error bars are inside simbols. The solid line is a linear best fit.

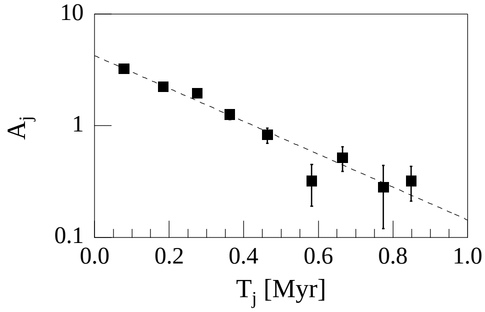

Fig. 3. A semi-logarithmic plot of the peak height A_j plotted versus the peak position T_j. The solid line is an exponential best fit.

moving box along with a good stability of the results. Although all the data set was considered to estimate the RTDF we limited our study to those polarity intervals that last less than 1 Myr. We underline that only the 5 Figure 1 displays the normalized RTDF $P(\delta t)$ of the polarity time intervals δt. The $P(\delta t)$ shows a series of decreasing and nearly equally spaced peaks. Although we cannot exclude other physical processes, we believe that this feature of the RTDF might be due to a stochastic synchronization process as in SR. To locate the position of each peak (T_j) and the height (A_j) we have decomposed the RTDF using a superposition of 9 Gaussian functions.

The position T_j of each peak as a function of the odd numbers is reported in Figure 2. As clearly shown in this figure T_j scales linearly with odd numbers defining a characteristic time scale $T_\Omega \sim 0.1$ Myr (see Eq. 1). This time scale is in good agreement with previous findings of a $\sim$ 100 kyr periodicity in the geomagnetic intensity and inclination, suggesting a possible link between geomagnetic polarity reversals and the variation of the Earth's orbital eccentricity.

In Figure 3 we report the behaviour of the peak height (A_j) versus its position T_j on a semi-logarithmic plot. Peak heights follows an exponential law characterized by a decaying time scale $T_k = (310 \pm 20)$ kyr. This time scale agrees the current mean polarity residence time and is a measure of the spontaneous transition rate (Hoyng et al. 2002).

3. Summary and conclusions

In this work we analysed the possibility that the geomagnetic polarity reversal phenomenon may be read in terms of a SR process. To investigate this possibility, we carefully analysed the position and the height of each peak of the pdf of the polarity time intervals. We found that the peak position scales linearly with odd numbers, identifying a characteristic period $T_\Omega \sim 100$ kyr. Moreover, peak heights, which are a function of peak positions, follow an exponential law and defining a new characteristic time scale $T_k \sim 320$ kyr. These results suggest that geomagnetic polarity reversals may be really a SR phenomenon involving two competing characteristic time scales T_Ω and T_k related to an external nearly periodic driving and to spontaneous transition rate, respectively. The possibility to consider the geomagnetic polarity reversal phenomenon in terms of a SR process is corroborated by a new model of geodynamo proposed by Hoyng et al. (2001), where the geodynamo is essentially represented as a bistable oscillator. The presence of a bistable system is, indeed, one of the key elements of the SR. Under this hypothesis, we can suppose that the geomagnetic polarity reversals are a consequence of a cooperative nonlinear phenomenon in which a weak external period signal nonlinearly couples to a spontaneous internal noise. We retain the weak external periodic signal can be identified with the periodic variation of the Earth's orbital eccentricity that is characterised by a period $T_e \sim 100kyr$. Moreover, as well known, one of the possible effects of the periodic variations of the terrestrial orbital eccentricity might be a change of the Earth's rotation rate. That can be a consequence of different mechanisms involving gravitational and orbital effects. Although both the possible mechanisms are very slight in magnitude, their effects could be enhanced by the Earth's intrinsic random fluctuations of the dynamo $\alpha-$effect, thus perturbing the conditions at the core-mantle boundary. We wish to remark that the alone orbital perturbations are not able to affect in considerable manner the topology of the magnetic and velocity pattern at the core mantle boundary and to cause polarity reversal. However, the situation should be reconsider in the case of a SR phenomenon where it is the nonlinear coupling between a slight modulating signal and the intrinsic spontaneous fluctuations that may produce a stochastic synchronisation phenomenon. Furthermore, it is common opinion that changes of the core mantle boundary conditions may induce topological rearrangements of the fluid outer-core motions (McFadden and Merrill 1995) that could trigger magnetic field reversal. Thus, the nonlinear coupling between the internal spontaneous fluctuations of the velocity and magnetic topology and the weak modulation of the core mantle boundary conditions, as they may results from the aforementioned nearly cyclic mechanisms, could be the origin of the observed multi-peaked structure of the RTDF of polarity intervals.

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