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Statistical properties of solar X-ray flares

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Abstract. Statistical properties of soft X-ray emission bursts produced by solar flares are compared to statistics of energy dissipation in a shell model of Magnetohydrodynamic (MHD) turbulence. The distribution of waiting times between successive bursts is shown to display a power law tail both for soft X-ray flares and for energy dissipation burts in the MHD shell model, as already reported in previous papers. Moreover, the Probability Density Functions (PDFs) of soft X-ray intensity fluctuations are characterized by the presence of wide, non-gaussian tails. The shape of the PDFs is nearly unchanged as the timelag, used to calculate fluctuations, varies. A very similar behavior is found for PDFs of energy dissipation fluctuations in the shell model. We suggest that these results support the idea that solar flares could represent bursty dissipative events of MHD turbulence.

Key words. Sun: flares—MHD—Turbulence

1. Introduction

Statistical properties of solar flares have been extensively studied in the last decades by analyzing the frequency distributions of several parameters (e.g. peak count rate, total counts and flare duration) of soft X-ray and hard X-ray bursts produced by flares. It has been shown that these distributions are usually well represented by power laws, that is, $f(x) = Ax^{-\alpha}$, where $f(x)dx$ is the fraction of flares with the parameter x between x and $x + dx$, while the constants A and α can be obtained from least-squares fit to the data (see e.g. Datlowe at al. (1974); Dennis (1985); Crosby et al. (1993)).

In this context, much attention has been given to the idea, proposed by Parker (1988), that flares result from the superposition of small energy release events (*nanoflares*), produced by the dissipation of the current sheets arising spontaneously in the active regions of the solar corona as an effect of the continous random motions of the field line footpoints occurring at photospheric level. In particular, the Parker hypothesis stimulated Lu & Hamilton (1991) to propose an *avalanche model* (also called

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sandpile model), based on the concept of Self-Organized Criticality (SOC) (Bak et al. 1987). This model is able to reproduce the power law behavior of the probability distributions of flare burst parameters.

However, it has been recently recognized that the statistics of time intervals Δt between two successive bursts represents a critical issue for the goodness of avalanche models in explaining solar flare statistics. In fact, it has been shown (Wheatland et al. 1998; Boffetta et al. 1999) that these models give rise to an exponential waiting time distribution (WTD), as a consequence of the fact that avalanches are independent, Poissonian events. On the other hand, recent analyses of hard X-ray (HXR) and soft X-ray (SXR) observations showed that the solar flare WTD clearly deviates from the exponential behavior expected from avalanche models (Wheatland et al. 1998; Boffetta et al. 1999). In particular, Boffetta et al. (1999), by using SXR data acquired by the *Geostationary Operational Environmental Satellites* (*GOES*), found that the WTD follows a power law $P(\Delta t) \propto \Delta t^{-\beta}$, with $\beta \simeq 2.4$, for waiting times greater than a few hours. Lepreti et al. (2001) extended this analysis, by showing that the sequence of SXR bursts is not consistent with a local Poisson process and that the observed WTD is well described by a Lévy function, which asimptotically gives a power law. These results indicate that waiting times are statistically self-similar and suggest the presence of long-range correlations in the flaring process.

To the aim of investigating in more detail the physical origin of solar flare statistics, in this work we will analyze, besides the waiting time distribution, the scaling behavior of the Probability Density Functions (PDFs) of SXR intensity fluctuations. As a comparison, the same analysis will be performed on fluctuations of energy dissipation rate in a shell model of MHD turbulence.

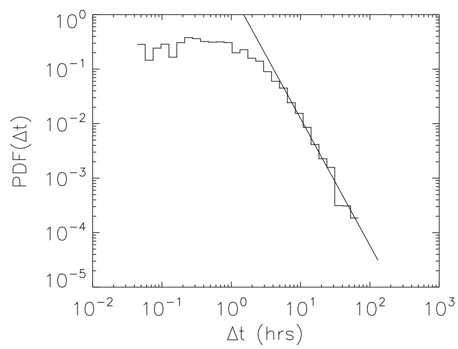

Fig. 1. Distribution of waiting times between successive soft X-ray flares. The solid line represent a power law with an exponent $\beta = 2.33$.

2. Analysis of flare soft X-ray bursts

In this paper, the statistical properties of solar flares are investigated by means of SXR data acquired by the *GOES* 10 satellite in the 1-8 Å band, during the time interval between 1998 August 1 and 2000 July 29. The SXR flux $f(t)$ was measured with a sampling time of 1 minute. In order to calculate waiting times, bursts are defined as the time intervals during which the condition $f(t) \geq f_{th}$ is satisfied. The threshold is defined as $f_{th} = \langle f(t) \rangle + 3\sigma$, where the average and the standard deviation are calculated through an iterative procedure, excluding the bursts (Boffetta et al. 1999). The WTD obtained from our analysis is well represented by a power law for $\Delta t \gtrsim 5$ hrs, with an exponent $\beta = 2.33 \pm 0.16$ (see Fig. 1). This is in agreement with the results of previous works (Boffetta et al. 1999; Lepreti et al. 2001), despite the fact we used a dataset covering a shorter period and we selected flares with a different criterion.

In order to obtain further indications for flare statistics modeling, we analyze the scaling bahavior of SXR intensity fluctuations $\delta f_\tau = f(t+\tau) - f(t)$. This is done by calculating the PDFs of standardized fluctuations $\delta F_\tau = (\delta f_\tau - \langle \delta f_\tau \rangle)/\langle(\delta f_\tau -$

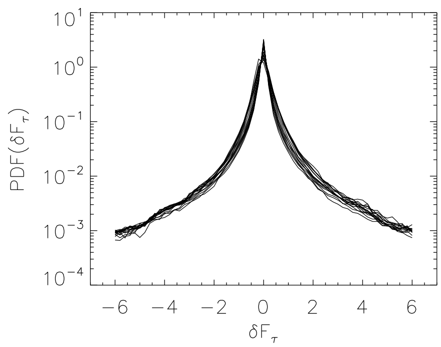

Fig. 2. PDFs of soft X-ray intensity fluctuations at different timelags τ in the interval 0.03 hrs $\leq \tau \leq 8.7 \times 10^3$ hrs

$\langle \delta f_\tau \rangle)^2\rangle^{1/2}$ (where brackets represent time averages) at different lagtimes τ. The shape of the PDF's of δF_τ, for 0.5 hrs $\leq \tau \leq 8.7 \times 10^3$ hrs, is shown in Fig. 2. It can immediately noted that the PDFs are strongly non-gaussian, due to the presence of high tails in the range of large increments, and that their shape does not change significantly, as τ varies.

3. Analysis of an MHD shell model

In this section, the same analysis tools described above will be applied to a shell model of MHD turbulence. This model is a dynamical system designed in order to reproduce the main features of nonlinear dynamics occurring in MHD turbulence (Boffetta et al. 1999), and is built up through the following main steps. The wavevector space is divided in discrete shells of radius $k_n = 2^n k_0$. Two complex dynamical variables $u_n(t)$ and $b_n(t)$, representing, respectively, velocity and magnetic field increments on an eddy of scale $l \sim k_n^{-1}$, are assigned to each shell. The evolution equations for $u_n(t)$ and $b_n(t)$ are obtained by introducing general quadratic couplings between neighbouring shells, and imposing the conservation of MHD ideal invariants. In this way, the following set of nonlinear Ordinary Differential Equations can be obtained:

$$\frac{du_n}{dt} = -\nu k_n^2 u_n + f_n + T_n(u_n, b_n)\,, \qquad (1)$$

$$\frac{db_n}{dt} = -\mu k_n^2 b_n + G_n(u_n, b_n)\,, \qquad (2)$$

where where ν and μ are the kinematic viscosity and the resistivity respectively, f_n is an external forcing term, T_n and G_n are the nonlinear quadratic terms (Boffetta et al. 1999).

The attempt to reproduce solar flare statistical properties through an MHD shell model is based on the idea that flares could be identified with energy dissipation bursts in MHD turbulence. The energy dissipation rate $\varepsilon(t)$ is given by

$$\varepsilon(t) = \nu \sum_n k_n^2 |u_n|^2 + \eta \sum_n k_n^2 |b_n|^2\,. \qquad (3)$$

Time series $\varepsilon(t)$ can be obtained from numerical simulatons of the model (Boffetta et al. 1999), and dissipation bursts can be found through the condition $\varepsilon(t) > \varepsilon_{th}$, where ε_{th} is a suitable threshold value for $\varepsilon(t)$.

Boffetta et al. (1999) already showed that dissipative bursts in the MHD shell model reproduce the statistical properties of X-ray solar flares, that is, the power law distributions for peak flux, total energy, durations and waiting times. The threshold chosen in the present paper to select dissipation bursts and calculate waiting times is given by $\varepsilon_{th} = \langle \varepsilon(t) \rangle + 3\sigma$ where $\langle \varepsilon(t) \rangle$ and σ are calculated in the same way as for *GOES* SXR data. The waiting time distribution (see Fig. 3) is characterized by the presence of a power law tail with a scaling exponent 2.07 ± 0.10.

As in the case of *GOES* SXR data, we also calculated the PDFs of standardized incrementss of the energy dissipation rate at different lagtimes τ, that is, $\delta E_\tau = (\delta\varepsilon_\tau - \langle \delta\varepsilon_\tau \rangle)/\langle(\delta\varepsilon_\tau - \langle \delta\varepsilon_\tau \rangle)^2\rangle^{1/2}$, where $\delta\varepsilon_\tau = \varepsilon(t+\tau) - \varepsilon(t)$. The PDFs of δE_τ at different lagtimes τ are shown in Fig. 4. It can be seen that the PDFs are non-gaussian

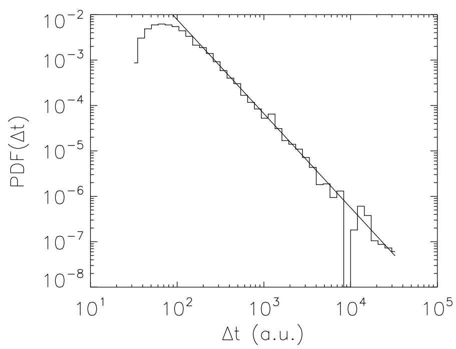

Fig. 3. Distribution of waiting times between successive dissipative bursts in the MHD shell model. The solid line represent a power law with an exponent $\beta = 2.07$.

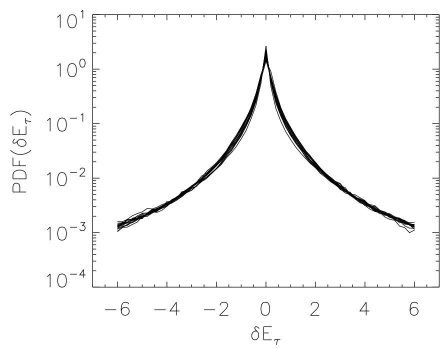

Fig. 4. PDFs of energy dissipation fluctuations, for the MHD shell model at different timelags τ.

tails, displaying strong tails at large increments, and don't change their shape significantly as τ varies. This behavior is in good agreement with the results obtained from the analysis of SXR data.

4. Conclusions

In this paper, we performed a statistical analysis of the soft X-ray bursts produced by solar flares and compared the results with a shell model of MHD turbulence. As already evidenced in previous papers (Boffetta et al. 1999; Lepreti et al. 2001), we point out that the waiting time distribution displays a power law tail, both for flare SXR bursts and for energy dissipation bursts in the MHD shell model. Besides the waiting time distribution, we investigated the scaling behavior of SXR intensity fluctuations. We showed that the PDFs of these fluctuations are characterized by the presence of wide, non-gaussian tails and have the same shape at different timescales. A very similar behavior was found for PDFs of energy dissipation fluctuations in the MHD shell model. In our opinion, these results suggest that solar flares could represent bursty dissipative events of MHD turbulence, as proposed by Boffetta et al. (1999). Following this idea, the power law behavior of the WTD can be related to temporal correlations produced by the nonlinear dynamics occurring in the system, while the non-gaussian PDFs can be interpreted as the result of strong events accumulating on the dissipative scale through the occurrence of nonlinear interactions.

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