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Oscillations of the Sun: insights and challenges for the future

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Abstract. The helioseismology, the study of solar oscillations, has proved to be a unique opportunity for investigating the internal structure and dynamics of the Sun. We present the status of helioseismic studies and comment on prospects for future investigations through the possibility of getting ever more precise helioseismic observations from ground and space and developing new techniques of analysis to reconstruct the complete picture of the Sun and solve the most discussed open questions in solar physics.

Key words. Sun's interior – pulsations

1. Introduction

Some forty years ago periodic motions, with periods around 5-min, were detected at the Sun's surface by Leighton et al. (1962). After various interpretations based on the local nature of these waves, Ulrich (1970) and independently Leibacher & Stein (1971) proposed a new interpretation of the oscillatory character of motions in terms of acoustic standing waves permeating the whole body of the Sun. The incontrovertible proof that this interpretation of solar 5-min oscillations was the correct one came five years later, thanks to the spectroscopic observations of Deubner (1975) who first was able to obtain a $k_h - \omega$ diagram which reflected the predictions of Ulrich and Leibacher & Stein. The demonstration of the normal mode character of solar oscillations constituted the births of the helioseismology.

Since Deubner's discovery, more than 10000 acoustic modes, with related splittings, were identified with an average accuracy in frequency of the order of $10^{-4} - 10^{-5}$, which have permitted a deep knowledge of the internal structure and dynamics of the Sun, not imaginable 20 years ago.

The observed solar five-minutes oscillations correspond to propagation of standing acoustic waves maintained by pressure forces, which form the class of the p modes, and to standing surface gravity waves, maintained by gravity, known as f

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modes. In addition, we should mention the internal gravity waves, g modes, with very high amplitude in the core, where the p modes are not so sensitive. Unfortunately, although claims for detection of g modes have been made (Gabriel et al. 1998), it has not yet confirmed that they are really excited in the Sun.

The basic concepts and the theory of solar oscillations can be found in the classical book by Unno et al. (1989), and recent developments in the review on helioseismology by Christensen-Dalsgaard (2002).

2. Helioseismic investigation

The goal of helioseismology is to infer the internal properties of the Sun and to understand the physical mechanisms which govern the behaviour of our star. This can be pursued by two different strategies: i)"global" helioseismology, based on analysis of normal mode frequencies, which reveals radial and latitudinal variations of the global properties of the solar structure and dynamics of the Sun; ii) "local" helioseismology, based on inversion of travel times of acoustic waves which propagate between different points on the solar surface through the interior, provides three-dimensional maps of the sound speed and flows in the upper convection zone, to probe local inhomogeneities in the sub-surface and surface layers.

Obviously, the results of the inferences depend on the mode selection $i \equiv (n, l)$ with radial order n and harmonic degree l, and on the observational errors which characterize the mode set ($i = 1, \ldots, M$) analyzed.

3. Global helioseismology

3.1. Inference of the sound speed

Far-reaching inferences about the global properties of the Sun have been obtained by applying inversion techniques to observations of frequencies of acoustic modes of oscillations. The first significant results concerning the application of the inversion technique to the Sun were obtained in 1985 by Christensen–Dalsgaard et al. (1985), who produced the sound speed profile in the interior of the Sun and determined for the first time the location of the base of the convection zone.

Since then, several efforts have been made for inverting data in order to test the correctness of the standard models in view of the improvements accomplished in the description of the relevant physics. A significant progress, in particular, has been achieved with the inclusion of diffusion of helium and heavy elements at the base of the convective zone (Basu et al. 1996).

The results, so far, have shown that the solar structure is remarkably close to the predictions of the standard solar model and, recently, that the near-surface region can be probed with sufficiently high spatial resolution as to allow investigations on the equation of state and solar envelope helium abundance.

Figure **??** shows the relative squared sound-speed differences between the Sun and two recent standard models, as results of the inversion of the data set by Basu et al. (1997), obtained in 1996 by the SOI/MDI instrument on board the SOHO satellite which includes modes with $l < 100$ and in particular very accurate frequencies of low harmonic degree, which allow to resolve the solar core. The two reference models used are the latest GARching SOlar MOdel (GARSOM5) described by Schlattl (1999) which employs a recent version of the OPAL2001 equation of state (Rogers 2001), and the Model S by Christensen-Dalsgaard et al. (1996) which employs the OPAL96 (Rogers et al. 1996) equation of state. These models are built on standard physical inputs, and assume full particle diffusion for hydrogen, helium and metals.

The differences between the Sun and the models prediction are extremely small, except below the base of the convection zone ($0.71\,R_\odot$) where the theory fails to correctly describe the turbulent convection. However, it can be noticed that the discrep-

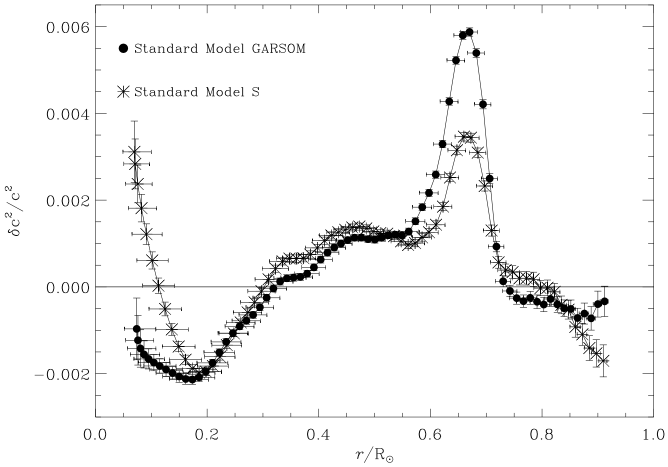

Fig. 1. The relative squared sound-speed difference between the Sun and two standard solar models: model S by Christensen-Dalsgaard et al. (1996) and model GARSOM5 (Schattl 1999). The differences, in the sense (Sun) - (model), are obtained by inversion of a set of data with accurate measurements of low degree modes (Basu et al. 1997). The vertical error bars correspond to the standard deviations based on the errors in the mode sets, whereas the horizontal bars give a measure of the localization of the solution.

ancy below the base of the convection zone is smaller for model S.

The structure of the core is still quite uncertain since the few modes with the lowest harmonic degree that are able to penetrate towards the centre, sample the core for a relative short time because of the large sound speed there. Figure **??** shows that in the core the model GARSOM5 is in better agreement with the Sun, than the model S.

The solar envelope can be probed with sufficiently high spatial resolution through inversion of high-degree acoustic modes ($l \geq 100$), which sound the region below the solar surface. Unfortunately high degree modes appear strongly affected by uncertainties of the surface layers. In order to suppress these uncertainties, an unknown function F_{surf}, the so-called surface term, is usually added to the equation governing helioseismic inversions (Dziembowski et al. 1990). The appropriate mathematical expression for F_{surf} is traditionally a function of the frequency alone $F_{\mathrm{surf}} \simeq f(\omega)$. However, as it was demonstrated by Di Mauro et al. (2002), when modes with high harmonic degree $l > 100$ are used, the surface term requires the explicit dependence on the harmonic degree $F_{\mathrm{surf}} = f(\omega, \omega/L)$, where $L = l + 1/2$.

The helioseismic results shown in Fig. **??** have been obtained by Di Mauro et al. (2002) by inverting helioseismic data which include high-degree modes ($l < 1000$), obtained in 1996 by the SOI/MDI instrument on board the SOHO satellite (Rhodes et al. 1998). Here the Model S (Christensen-Dalsgaard et al. 1996) was used as a reference model. Figure **??** shows that high-degree modes enable inference of properties very close to the solar surface, through the HeII ionization zone and also part of the HeI ionization zone. Below the photosphere, for $r \geq 0.95\,R_\odot$, high-degree modes reveal a large discrepancy between the model and the observed Sun, even considering higher-order asymptotic terms in F_{surf}.

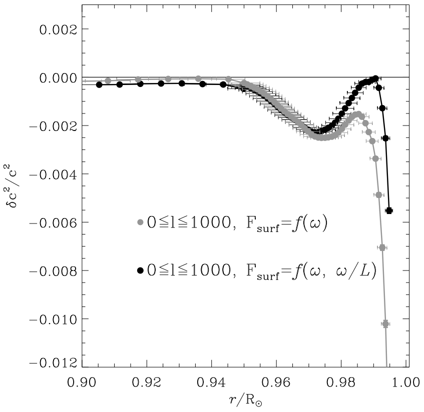

Fig. 2. The relative squared sound-speed difference between the Sun and the standard solar model S (Christensen-Dalsgaard et al. 1996), in the sub-surface layers, as obtained by inversion of a set of data with high-degree modes (Rhodes et al. 1998). The solutions are obtained by considering the surface term $F_{\rm surf} = f(\omega, \omega/L)$ suitable for the treatment of high degree modes and a traditional surface term which depends on frequency alone $F_{\rm surf} = f(\omega)$.

3.2. Equation of state and composition

The solar plasma is almost an ideal gas, and the first adiabatic exponent Γ_1, the partial logarithmic derivative of pressure with respect to density at constant specific entropy, is therefore close to 5/3 in most of the interior. It deviates from this value in the zones of hydrogen and helium ionization, near the surface. Therefore, inversions of helioseismic data can be used, in particular, to study the equation of state and to probe the helium abundance in the solar envelope.

Here we will compare the results obtained by using two reference models – Model $\mathrm{S_{OPAL}}$ and Model $\mathrm{S_{MHD}}$ – by Christensen-Dalsgaard et al. (1996), which use respectively the OPAL (Rogers et al. 1996), and the MHD (Mihalas et al. 1988), equations of state.

Basu & Christensen-Dalsgaard (1997) and Di Mauro (2001) by using only low and intermediate-degree modes showed that it is difficult to judge the significance of the differences between the two equations of state. Furthermore, it appeared clear that in the central core Γ_1 deviates from 5/3, probably due to relativistic effects (Elliott & Kosovichev, 1998).

Figure **??**, taken from Di Mauro et al. (2002) shows the intrinsic differences in Γ_1 between the Sun and the two equations of state considered (OPAL and MHD), as obtained by inversion of the set data with high degree modes (Rhodes et al. 1998). As it was noticed by Di Mauro et al. (2002), the OPAL equation of state describes more accurately the plasma conditions below $0.97\,R_\odot$. The MHD model gives a more accurate description in the layers with $0.97\,R_\odot \leq r \leq 0.99\,R_\odot$, while for $r \geq 0.99\,R_\odot$ the differences between the two equations become very small.

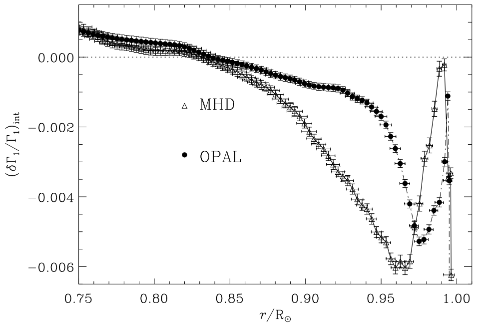

Fig. 3. The intrinsic differences of the adiabatic exponent Γ_1 between the Sun and the OPAL (filled circle) and MHD (open triangles) equations of state, obtained by an inversion of a set of data by Rhodes et al. (1998), which includes high-degree modes.

The first seismic measures of the envelope helium abundance $Y_{\rm e}$, obtained by Christensen–Dalsgaard et al. (1996) reported values between 0.24 and 0.25, that were significantly smaller than the abundance estimated by the calibration of the helium initial abundance on the standard solar model. Dziembowski et al. (1991) pointed out that the difference was in rough agreement with what expected by the effect of gravitational settling of helium and heavy elements, as calculated by Cox et al. (1989). So, the settling process is included in all the most accurate standard solar models nowadays. However, it appeared clear that the determination of the solar helium abundance is sensitive to the equation of state employed in the reference model. Recently, Di Mauro et al. (2002) have obtained a value of the helioseismic helium abundance in the convection zone $Y_{\rm e} = 0.2457 \pm 0.0005$ by using the MHD equation of state, and a value of $Y_{\rm e} = 0.2539 \pm 0.0005$ by considering the OPAL equation of state, both in quite good agreement with earlier findings.

3.3. Solar rotation

It is well known, and easily observed at the photosphere, that the Sun is slowly rotating. The rotation breaks the spherical symmetry and splits the frequency of each oscillation mode of harmonic degree l into $2l + 1$ components, known as "splittings". By applying standard perturbation theory to eigenfrequencies, it can be shown that the rotational splitting for each mode is directly related to the rotation rate $\Omega(r, \theta)$ inside the Sun, where r is the radius and θ is the co-latitude. The dependence of the splittings on angular velocity can be used in a 2-dimensional inverse problem to probe the Sun's internal differential rotation.

The most interesting results on the internal angular velocity of the Sun have been obtained in the last few years (e.g. Di Mauro et al. 1998) , thanks to the observational accuracy reached with the uninterrupted series of observations by the GONG network and by the SOI-MDI instrument on board the SOHO satellite (Schou 1998).

The results have shown that the latitudinal differential rotation observed at the surface persists throughout the convection zone, while the radiative interior rotates almost rigidly at a rate of about 430 nHz. At low latitudes, the angular velocity, through the largest part of the convection zone, decreases inwards, while at high latitudes increases inwards. The near-surface behaviour is consistent with the observed surface rotation rate. The tachocline, the transition layer from latitudinally-dependent rotation to nearly independent rotation, is of very considerable dynamical interest. In fact, it is thought that the global dynamo behaviour, responsible for the solar magnetic cycle, rises from strong toroidal magnetic fields generated by rotational shear in this thin region. The tachocline appears to be located mostly in the radiative zone at a quite sharp midpoint $r = 0.693\,R_\odot$, according to Corbard et al. (1998), and $r = 0.695\,R_\odot$ for Charbonneau et al. (1999). This thin layer has a thickness that changes with the latitude, with a minimum value at the equator of $(0.0039 \pm 0.0013)\,R_\odot$ (Charbonneau et al. 1999). Recently, Howe et al. (2000) have found evidence that the rotation rate near the base of the convective envelope shows time variations, with a period of the order of 1.3 yr, at low latitudes. Such variations occur above and below the tachocline and appear to be more pronounced near the equator and at high latitudes. Physical mechanism that might be responsible for this behaviour was discussed by Thompson (2001).

In order to infer accurately the rotation in the deepest interior, it is necessary to invert a set of data which includes accurate splittings of the lowest degree modes ($l = 1 - 4$). The data sets available for this purpose are obtained from the ground-based networks BiSON (Chaplin et al 1996), IRIS (Fossat 1991) and GONG (Harvey et al. 1996) and the GOLF-SOHO (Gabriel et al. 1995) instrument. Unfortunately these sets of data are not in mutual agreement and give conflicting results of inversion in the core. In fact, the independent sets of observations obtained by IRIS, GONG, GOLF lead to the conclusion that the Sun's core rotates slightly faster than the surface, in contradiction with the BiSON's data inversion that indicates a central angular velocity even slower than the surface polar angular velocity (see Di Mauro et al. 1998, Chaplin et al. 1999). Thus, the kinematics in the core remains quite uncertain, with a disagreement that might derive from the different procedures of data analysis employed.

3.4. Solar seismic radius

The helioseismic investigation of the solar radius is based on the principle that the frequencies of the f modes of intermediate angular degree depend primarily on the solar surface radius. In the 1997, Schou et al. (1997) succeed for the first time, in obtaining an helioseismic determination of the solar radius by using high-precision measurements of frequencies of the f modes of the Sun, obtained from the MDI experiment on board the SOHO spacecraft. They determined that the seismic radius is about 300 km smaller than the standard solar radius as determined from limb darkening, as it was confirmed by Antia (1998).

Dziembowski et al. (1998) by analyzing a long time-series of MDI f-mode frequencies, found that the seismic radius varies with time in correlation with the magnetic activity.

Only recently, Dziembowski et al. (2001) were able to explain the connection between the standard solar radius and that inferred from f-modes, and confirmed the previous results on the contraction of the Sun's outer layers during the rising phase of the solar magnetic activity, with a very small effect on the photospheric radius.

3.5. Solar asphericities

The asymmetric part of the fine structure in the p-mode spectrum of solar oscillations varies in a systematic way through the solar cycle (e.g. Kuhn et al. 1988).

It is evident that the changes are associated with the surface temperature bands reported by Kuhn et al. (1996) and with solar surface magnetic variations (Woodard & Libbrecht 1991).

The origin of this behaviour, as well as the temporal variation of the frequencies is still ambiguous, but it appears clear that all these changes are consistent with a near surface perturbation.

Dziembowski et al. (1998) have studied the behaviour of the even splitting coefficients of p modes, obtained by observations covering almost all the period during the past 11 years cycle. Their conclusion was that the Sun assumes a shape which varies from simply oblate to complicated asphericity according to the magnetic cycle. This interesting conclusion has been confirmed by Howe et al. (1999), which analyzed data obtained by the GONG network during the period 1995-1998.

4. Local Helioseismology

So far we have considered only global standing modes of solar oscillation, whose frequencies reflect the properties of that part of the Sun through which the waves travel. It is clear that since global modes extend over all longitudes, they cannot provide information about longitudinal variation of the characteristics of the structure and of the rotation of the Sun.

Time-distance helioseismology, introduced by Duvall et al. (1993), has yielded numerous exciting insights about the interior of the Sun. This technique, which gives information about travel times for wave packets moving between any two points on the solar surface, is an important complement to global-mode helioseismology, as it is able to probe subsurface structure and dynamics in three dimensions. Early investigations of this nature demonstrated that sunspots are strong absorbers of incident p modes (Braun et al. 1987), and the subsurface structure of active regions can be probed by means of helioseismology.

Some of the main results concern flows and wave-speed perturbations underneath sunspots (e.g. Zhao et al. 2001), largescale subsurface poleward flows (Giles et al. 1997), and supergranulation flows (Duvall & Gizon 2000).

5. Future perspectives

Helioseismology, through the very accurate identification of oscillation frequencies of acoustic and fundamental modes, has clearly demonstrated that the standard solar models reproduce the behaviour of the Sun with remarkably accuracy, consistent within 1 %.

Despite such overall success, this discipline has not yet exhausted its resources, since helioseismic results clearly suggest further refinements of the solar models.

The solar models are based on several, perhaps questionable, assumptions about the physical properties of matter in stars, such as the equation of state, the opacity and the rates of nuclear reactions. In particular, the computation of models involves a number of simplifying hypotheses such as the treatment of convection, generally approximated by mixing-length theory, while the dynamical effects of the turbulent pressure are neglected. Therefore, the detailed structure of the convective zone and near-surface regions is still quite uncertain, and remain substantial ambiguities in modelling convective effects and thermodynamic properties, as well as in the treatment of non-adiabatic effects on the oscillations.

The attempts to understand the solar core conditions, up to now, have been contradictory too. In fact p modes (as opposed to g modes) are not very sensitive to the core of the Sun. This indicates the necessity of using more accurate low degree p-mode data and continue to investigate on the possible presence of g modes.

In addition, there is still much work ahead for getting a detailed understanding of the Sun's rotation. Some rotational features like, for example, the temporal changes which occur near the base of the convective envelope have not yet been explained.

Finally, by studying the connection between the seismic and the global characteristics of the Sun, the challenge is to find the reason for the correlation between the variation of the Sun's shape and the magnetic solar cycle.

Ever more accurate helioseismic observations from ground and space can help us to reconstruct the complete picture of the Sun and, finally, to solve the most discussed open questions in solar physics such as the history of the Sun's angular momentum and the solar cycle generation mechanism.

Recently, a new window has been opened on the astrophysics research: the possibility to study and to understand the structure and dynamics of other stars by applying the tools and the well developed techniques used in helioseismology. In fact, the success of helioseismology has spurred investigators to extend this diagnostic method to other stars which may show multi-mode pulsations. Up to now, the seismological study of pulsating stars, known as "asteroseismology", has been hindered by the problem of mode identification since the oscillation amplitudes observed on the Sun (a few parts per million in flux) are too small to be detected in other stars with ground-based telescopes. To reach the required sensitivity and frequency resolution, several space experiments will soon be devoted to the measurements of stellar oscillations. Thus, it is evident that asteroseismology represents the next step in the evolution of the helioseismic research.

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