ABSTRACT

The expansion center model (presented by the author in paper I and II for the 1999 SAIt meeting in Naples - Mem. Soc. Astr. It. Vol. 71, N. 4, 2000), as expressively referred to the Milky Way position within the framework of a spherical homogeneous and isotropic inner Universe radially decelerated towards the expansion center, allows to picture a local Universe outline according to Dirac’s theory (1937), the so-called large numbers hypothesis (LNH), which is based on possible simple relations between cosmological and microphysical quantities.

Among the most significant results are to cite: $t_0 = \frac{1}{3H_0 r_c}$ and $G \propto t^{-1}$.

1. OUR EPOCH $t_0$ by the ECM

Let us consider two fundamental relations carried out by the expansion center model (ECM hereafter), those obtained in the section BY A SIMULATION of the paper I (1), as expressively referred to the Galaxy Hubble function ($H_{MW}$) and to the Galaxy radial distance ($R_{MW}$) from the huge void center. They are:

$$H_{MW} = H_0 \left(1 - \frac{3H_0 r_c}{c}\right)^{-1} \quad (1)$$

$$R_{MW} = R_0 \left(1 - \frac{3H_0 r_c}{c}\right)^{1/3} \quad (2)$$
Remember that \( r (r \leq \frac{c}{3H_0}) \) is the light-space run with speed \( c \) by light to reach us from the source; working in Mpc units and seconds, we have:

\[
\frac{dr}{dt} = -c_{\text{Mpc}/s} = -\frac{1_{\text{Mpc}}}{N_s}
\]  

(3)

\[
r = \int_0^r dr = -c_{\text{Mpc}/s} \int_{t_0}^t dt = -c_{\text{Mpc}/s}(t - t_0) = \frac{t_0 - t}{N}
\]  

(4)

where \( N (\equiv 1.029 \times 10^{14} \text{ s/Mpc}) \) is the number of light seconds corresponding to 1 Mpc.

At the same time, eq. (2), in Hubble units, gives immediately our epoch (just the one of Dirac’s theory (1937)-cf. Coles & Lucchin, 1995, p. 54), as

\[
R_{\text{MW}} = 0 \Rightarrow r_{\text{Mpc}} = \frac{c}{3H_0}
\]  

(5)

Such a \( r_{\text{Mpc}} \), once taken \( H_0 = 70 \pm 3 \) in paper II\(^{(2)}\) as derived by Sandage & Tammann 1975 data, gives the following age of the Universe expressed in time:

\[
(t_0)_s = N \frac{c}{3H_0} \Rightarrow (t_0)_{\text{years}} = (4.65 \pm 0.20) \times 10^9
\]  

(6)

2. MAIN FORMULAS ACCORDING TO DIRAC

At first one can show the following dimensionless equality

\[
1 - \frac{3H_0 r}{c} \equiv \frac{t}{t_0}
\]  

(7)

that results through the introduction of the (4) and (6) expressions as follows

\[
1 - \frac{3H_0}{c} \left( \frac{t_0 - t}{N} \right) = 1 - \frac{t_0 - t}{t_0} = \frac{t}{t_0}
\]  

(8)

Then eqs. (1) and (2) become directly

\[
H_{s^{-1}} = H_{0s^{-1}} \left( \frac{t_0}{t} \right)
\]  

(9)

\[
R_{\text{cm}} = R_{0\text{cm}} \left( \frac{t}{t_0} \right)^{1/3}
\]  

(10)
The previous different formulations of $H_{MW}$ and $R_{MW}$ have to be inserted into the velocity and deceleration formulas of our Galaxy, the ones derived in paper I as
\[ \dot{R}_{cm/s} = H_{s^{-1}} R_{cm} \] (11)
\[ \ddot{R}_{cm/s^2} = -2 H_{s^{-1}}^2 R_{cm} \] (12)

So the above formulas take the following formulations as functions of time:
\[ \dot{R}_{cm/s} = H_{0s^{-1}} R_{0cm} \left( \frac{t}{t_0} \right)^{-2/3} \] (13)
\[ \ddot{R}_{cm/s^2} = -2 H_{0s^{-1}}^2 R_{0cm} \left( \frac{t}{t_0} \right)^{-5/3} \] (14)

Eq. (10), holding the conservative law that mass does not change with time, furnishes the density law
\[ \rho(t) = \rho_0 \frac{t_0}{t} \] (15)

that, with (9), when inserted into the ECM density formula
\[ \rho \geq \frac{3 H_{s^{-1}}^2}{2 \pi G} \] (16)

together finally confirm the controversial Dirac result $G \propto t^{-1}$ (cf. Coles & Lucchin, 1995, p. 54) if it is the equality sign in (16) to be held. In fact we obtain
\[ G \geq \frac{3 H_{0s^{-1}}^2}{2 \pi \rho_0} \frac{t_0}{t} \] (17)

where the lower limit value for $G$ is referring to the ECM with only radial expansion, without any rotation.

Indeed all the previous results, in particular those represented by eqs. (6)(10)(13)(17), agree with Dirac’s theory (1937-1938), the so-called large number hypothesis (LNH), which is based on possible simple relations between cosmological and microphysical quantities.

3. THE VARIATION OF $G$
In the context of the ECM we wish briefly to refer to the problem of the $G$ variation. In fact, if one would take into consideration the presence of a hypothetical centripetal acceleration due to an undefined angular velocity $\dot{\theta}$, the Galaxy radial deceleration (12) towards the expansion center should be written as follows according to classical mechanics:

$$\ddot{R} = -2H^2 R = -\frac{4}{3}\pi \rho GR + R\dot{\theta}^2$$  \hfill (18)

Consequently, after the introduction of the (9) and (15) formulations, it results

$$G(t) = \frac{3H_0^2}{2\pi \rho_0} \left( \frac{t_0}{t} + \frac{\dot{\theta}^2}{2H_0^2 t_0} \right)$$  \hfill (19)

which, derived with respect to time, gives

$$\dot{G}(t) = \frac{3H_0^2}{2\pi \rho_0} \left( -\frac{t_0}{t^2} + \frac{\dot{\theta}^2}{2H_0^2 t_0} + \frac{\ddot{\theta} \dot{t}}{H_0^2 t_0} \right)$$  \hfill (20)

The usual ratio $\dot{G}_0/G_0$, through the above eqs. (20) and (19) applied to the Galaxy at our epoch $t = t_0$, after fixing the position

$$\frac{\dot{G}_0}{G_0} t_0 = \varepsilon_0$$  \hfill (21)

leads to the equation

$$\dot{\theta}_0^2 = 2H_0^2 \left( \frac{1 + \varepsilon_0}{1 - \varepsilon_0} \right) - \frac{2\dot{\theta}_0 \ddot{\theta}_0 t_0}{1 - \varepsilon_0}$$  \hfill (22)

whose solution by points gives

$$\dot{\theta}_0 = nH_0 \quad \ddot{\theta}_0 = \xi(\varepsilon_0, n) \frac{H_0}{t_0}$$  \hfill (23)

with

$$\xi(\varepsilon_0, n) = \frac{1}{n} (1 + \varepsilon_0) - \frac{n}{2} (1 - \varepsilon_0) \leq 0 \quad (24)$$

Putting the above equation $\xi(\varepsilon_0, n)$ equal to zero means to find the limit value of $n$.
\[ n_t = \sqrt{2 \left( \frac{1 + \varepsilon_0}{1 - \varepsilon_0} \right)} \]  \hspace{1cm} (25)

for which it is \( \tilde{\theta}_0 = 0 \); (25) also says that it must be always

\[-1 \leq \varepsilon_0 < 1 \]  \hspace{1cm} (26)

Hence, the first problem to solve is to find a faithful value of \( \varepsilon_0 \).

At this regard, having already obtained \( t_0 \) in (6), we have to look for values of the ratio \( \dot{G}/G \), the ones computed experimentally through different techniques. A few results from the recent scientific literature are the following, starting with the historic data by Shapiro:

1) by I. I. Shapiro (1976) \( \left| \frac{\dot{G}}{G} \right| < 10 \times 10^{-11} \text{yr}^{-1} \);

2) by P. M. Muller (in J. V. Narlikar, 1993) \( \frac{\dot{G}}{G} = (-6.9 \pm 3.0) \times 10^{-11} \text{yr}^{-1} \);

3) by Van Flandern (in J. V. Narlikar, 1993) \( \frac{\dot{G}}{G} \sim (-6.9 \pm 2.4) \times 10^{-11} \text{yr}^{-1} \);

4) by 1983 space experiment (in J. V. Narlikar, 1993) \( \frac{\dot{G}}{G} = (0.2 \pm 0.4) \times 10^{-11} \text{yr}^{-1} \);

5) by Guenther et al. (1998) \( \left| \frac{\dot{G}}{G} \right| \leq 0.16 \times 10^{-11} \text{yr}^{-1} \);

6) by Reasenberg R. D. (1983) \( \left| \frac{\dot{G}}{G} \right| \leq (3 \pm 0.6) \times 10^{-11} \text{yr}^{-1} \);

7) by Damour et al. (1988) \( \left| \frac{\dot{G}}{G} \right| \leq (1.10 \pm 1.07) \times 10^{-11} \text{yr}^{-1} \);

8) by Garcia-Berro E. et al. (1995) \( \left| \frac{\dot{G}}{G} \right| \leq (1 \pm 1) \times 10^{-11} \text{yr}^{-1} \).

At this point the choice can be simply to adopt the average \( \left| \frac{\dot{G}}{G} \right| \lesssim 3.6 \times 10^{-11} \text{yr}^{-1} \).

Such upper limit is practically the same which follows from solar system measurements (\( \left| \frac{\dot{G}}{G} \right| \lesssim 3 \times 10^{-11} \text{yr}^{-1} \) in Michael Rowan-Robinson, 1996). Then the correct value, as approximative definitive order of magnitude of the variation of \( G \), results to be

\[ \frac{\dot{G}_0}{G_0} \sim -10^{-11} \text{yr}^{-1} \]  \hspace{1cm} (27)

So it is

\[ \varepsilon_0 \sim -0.05 \]  \hspace{1cm} (28)
and
\[ n_l \approx 1.345 \quad (29) \]

4. LOCAL UNIVERSE OUTLINE

The above limit value 1.345 for \( n \) represents the zero value for \( \xi(\varepsilon_0, n) \), that is to say \( \dot{\theta}_0 = 0 \). As it is very plausible to think to some angular deceleration, in presence of some angular velocity, now let’s try to picture a local Universe outline by points through the assumption of only 3 different values of \( n \), all corresponding to the same \( \varepsilon_0 \sim -0.05 \). From (18), being \( \dot{\theta}_0 = nH_0 \), the density formula becomes:
\[ \rho_0 = \frac{3H_0^2(2 + n^2)}{4\pi G_0} \quad (30) \]

Hence, by the obtained values of \( H_0(\approx 70Km \ s^{-1} \ Mpc^{-1}(\approx 2.27 \times 10^{-18} s^{-1})) \) and \( R_0(\approx 260 \ Mpc(\approx 8.0 \times 10^{26} cm)) \) of the paper II\(^{(2)}\), it results for the Galaxy at our epoch \( t_0(\approx 1.469 \times 10^{17} s) \) the series of numerical values, of \( \dot{R}, \ddot{R}, \dot{R}_0, \xi, \rho \), as listed in the table below:

<table>
<thead>
<tr>
<th>( t = t_0 )</th>
<th>( n = 1 )</th>
<th>( n = 1.345 )</th>
<th>( n = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{MW} )</td>
<td>( 1.82 \times 10^9 cm/s )</td>
<td>( 1.82 \times 10^9 cm/s )</td>
<td>( 1.82 \times 10^9 cm/s )</td>
</tr>
<tr>
<td>( \dot{R}_{MW} )</td>
<td>( -8.24 \times 10^{-9} cm/s^2 )</td>
<td>( -8.24 \times 10^{-9} cm/s^2 )</td>
<td>( -8.24 \times 10^{-9} cm/s^2 )</td>
</tr>
<tr>
<td>( R\dot{\theta}_{MW} )</td>
<td>( 1.82 \times 10^9 cm/s )</td>
<td>( 2.45 \times 10^9 cm/s )</td>
<td>( 3.64 \times 10^9 cm/s )</td>
</tr>
<tr>
<td>( \xi(\varepsilon_0, n) )</td>
<td>( +0.425 )</td>
<td>( 0.0 )</td>
<td>( -0.575 )</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>( 0.55 \times 10^{-28} g/cm^3 )</td>
<td>( 0.70 \times 10^{-28} g/cm^3 )</td>
<td>( 1.1 \times 10^{-28} g/cm^3 )</td>
</tr>
</tbody>
</table>

In conclusion it is even possible to try a direct computation of other extreme values, which the eqs. (10)(13)(14)(15)(17) seem to be able to give to \( \dot{R}, \ddot{R}, \dot{\rho}, G \) respectively, whether these formulas may be thought to hold also for \( t \to 0 \), always in reference to the Milky Way position with respect to the inner Universe and to its expansion center, at the epoch (as example) of one second after the Big Bang. In this hypothetical and only explorative case we find the following numerical results:

\[ R(t = 1 \text{ sec}) \approx 1.6 \times 10^3 \text{ lightyears} \quad (31) \]
\[ \dot{R}(t = 1 \text{ sec}) \approx 1.7 \times 10^{10} \text{ lightspeed} \quad (32) \]
\[ \ddot{R}(t = 1 \text{ sec}) \approx 0.66 \times \dot{R}_{t=1}/ \text{ sec} \quad (33) \]
\[ \dot{\rho}(t = 1 \text{ sec}) < 10^{-11} g/cm^3 \quad (34) \]
\[ G(t = 1 \text{ sec}) > 10^{17}G_0 \quad (35) \]
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