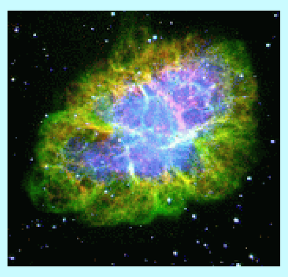

A SIMPLE AND EFFICIENT HIGH–RESOLUTION SHOCK–CAPTURING METHOD FOR RELATIVISTIC FLUID DYNAMICS

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Relativistic flows play an essential role in high energy astrophysics, and relativistic shocks are supposed to be the place where particle acceleration occurs. Among the astrophysical sources of such flows and shocks there are AGNs, and the related jets, the GRB fireball, and pulsar wind nebulae. We present an easy to implement multidimensional shock–capturing relativistic hydrodynamics (RHD) scheme. RHD numerical schemes are generally more expensive than their corresponding Eulerian version essentially for the complexity of the relation between conservative and primitive variables. Our scheme is based on third order CENO reconstruction and on averaged Riemann solvers which do not require characteristic decomposition. The scheme is efficient and robust, even in multidimensional simulations, and can cope with very high Lorentz factors, giving results comparable with those of more sophisticated methods.

The equations of special relativistic hydrodynamics may be cast in the 3D conservative form

$$\frac{\partial \bar{u}}{\partial t} + \sum_{i=1}^{3} \frac{\partial \bar{f}^{\,i}(\bar{u})}{\partial x^i} = 0$$

where

$$\bar{u} = [\rho\gamma,\ w\gamma^2 v^j,\ w\gamma^2 - p]^T$$

$$\bar{f}^{\,i}(\bar{u}) = [\rho\gamma v^i,\ w\gamma^2 v^j v^i + p\delta^{ij},\ w\gamma^2 v^i]^T$$

$$\gamma = (1-v^2)^{-1/2};\quad e = \rho + p/(\Gamma - 1);\quad w = e + p$$

The main features of our code are:

- ✰ *Point values rather than cell average are used, making the extension to 1D to 3D easier.*
- ✰ *No characteristic decomposition and accurate Riemann solvers are required, thus achieving simplicity and efficiency. This will be a crucial point for the MHD extension (work is in progress)*
- ✰ *Primitive variables are recovered by solving a single equation for the Lorentz factor iteratively, just once at each time step.*
- ✰ *Computing efficiency: all simulations have been run on a 1GHz PC. The 100^3 3D blast wave simulation takes just a few hours*

The time integration is performed with a TVD third order Runge–Kutta method. For every subcycle and for every direction the following steps are taken

1–primitive variables are recovered from conservative ones in every cell

2–primitive variables are reconstructed at cell interfaces to give a left (L) and right (R) state. The intrepolation is done separately on each variable with a CENO third order tecnique using min–mod (MM) or monotonized–centered (MC) limiters near discontinuities

3–fluxes are computed with one of the two approximate Riemann solvers

$$\bar{f}^{HLL} = \frac{\alpha^+\bar{f}^L + \alpha^-\bar{f}^R - \alpha^+\alpha^-(\bar{u}^R - \bar{u}^L)}{\alpha^+ + \alpha^-}$$

$$\bar{f}^{LLF} = \tfrac{1}{2}(\bar{f}^L + \bar{f}^R - max\{\alpha^+,\alpha^-\}(\bar{u}^R - \bar{u}^L))$$

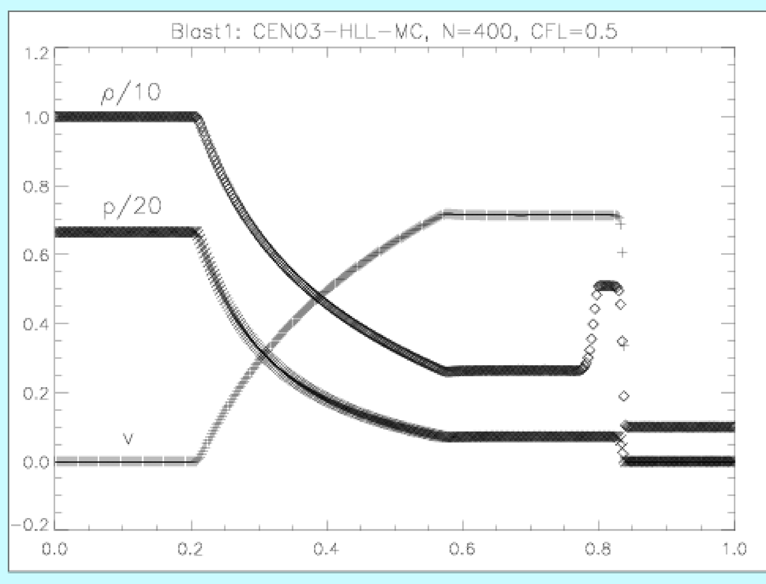
with

$$\alpha^\pm = max\{0, \pm\lambda^\pm(\bar{u}^L), \pm\lambda^\pm(\bar{u}^R)\}$$

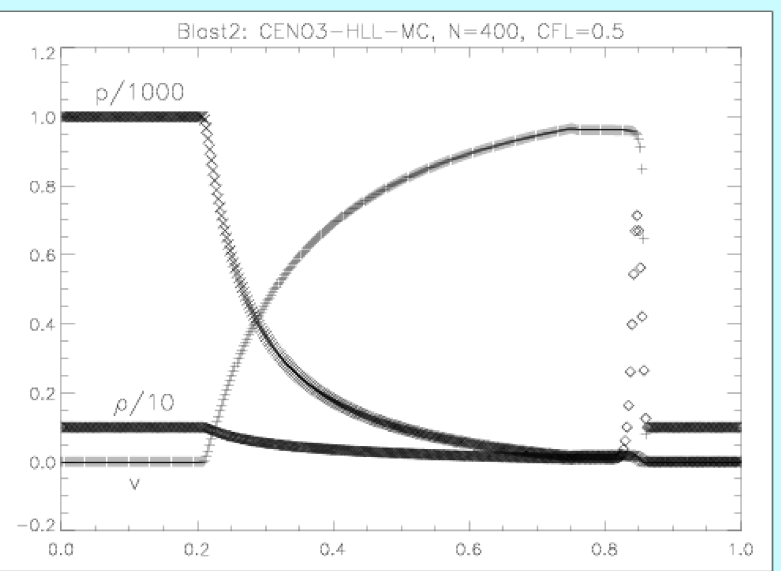
$$\lambda^\pm = \frac{v_{||}(1-c_s^2) \pm \sqrt{(1-v^2)(1-v_{||}^2 - v_\perp^2 c_s^2)}}{1 - v^2 c_s^2}$$

4–flux derivatives are reconstructed at cell–centers using another CENO routine to the same accuracy order

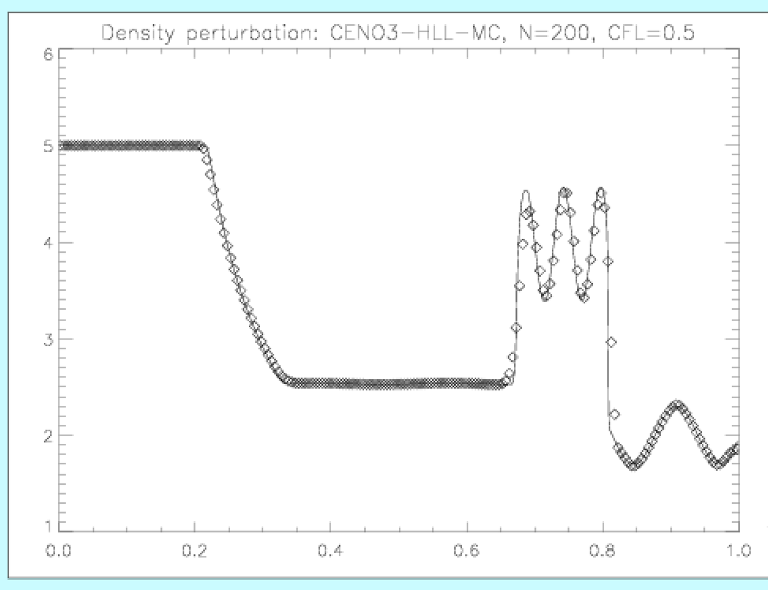
NUMERICAL RESULTS



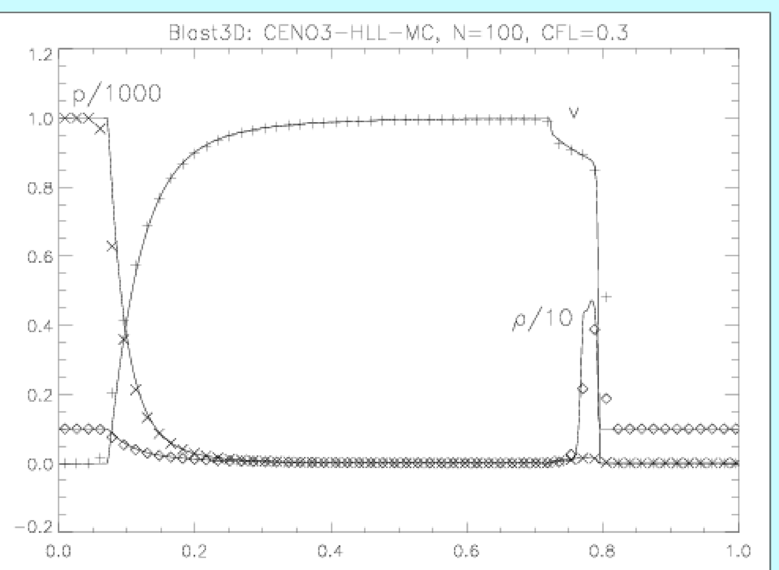
Blast wave 1:
(ρ,v,p) = (10,0,13.3)   x < 0.5
(ρ,v,p) = (1,0,10^{-6})   x > 0.5
the values of density, and velocity are well reproduced, there is only a little smearing in the position of the contact discontinuity



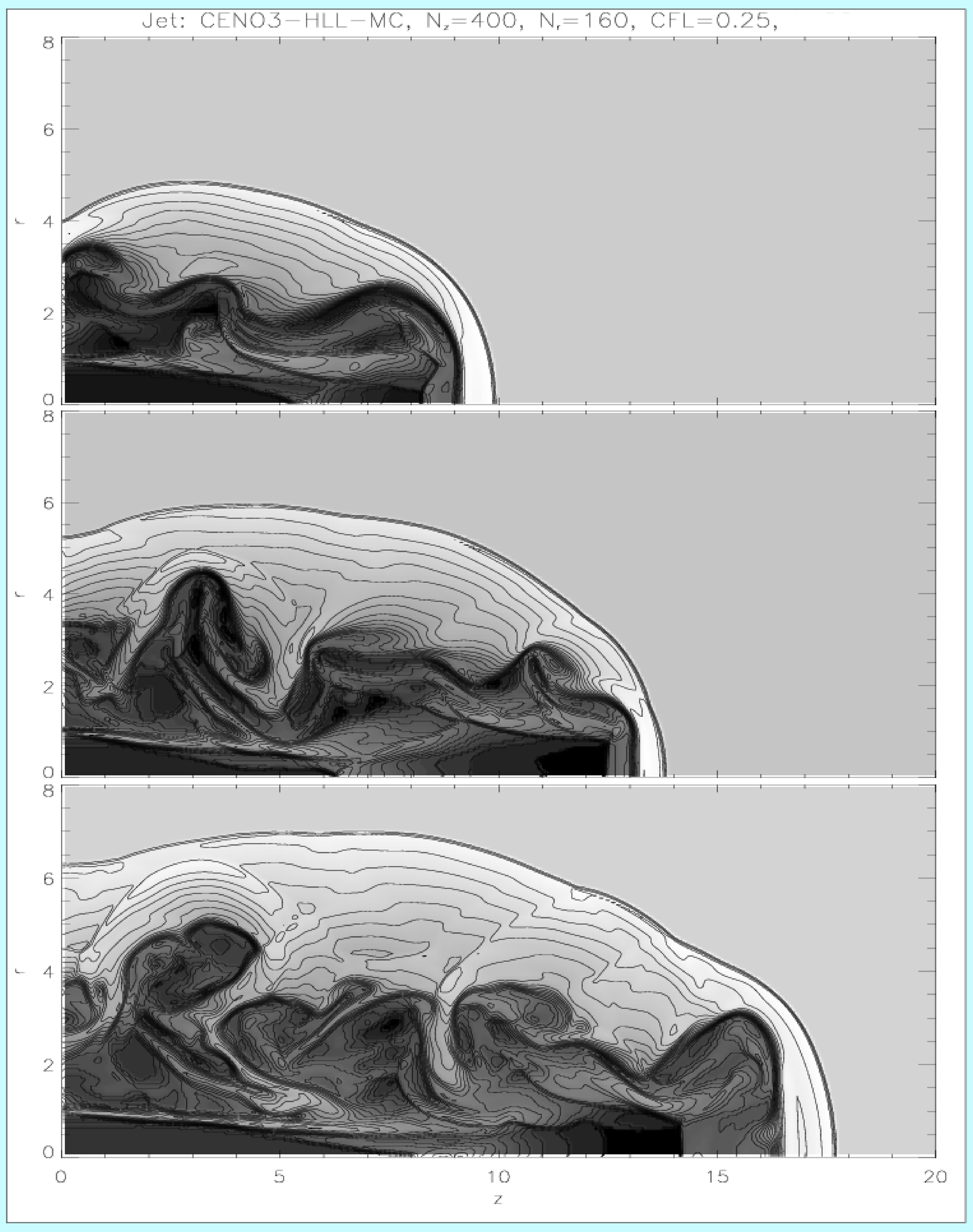
Blast wave 2:
(ρ,v,p) = (1,0,1000)   x < 0.5
(ρ,v,p) = (1,0,0.01)   x > 0.5
the peak is 70% of the true value, note the absence of oscillation and the sharp position of the front shock and back rarefaction wave.



Blast wave + density perturbation:
(ρ,v,p) = (5, 0, 50)   x < 0.5
(ρ,v,p) = (2 + 0.3 sin(50x), 0, 5)   x > 0.5
Solid line is the exact solution
This shows the accuracy of ENO schemes in treating both smooth and discontinuos fields



3D spherical blast wave:
(ρ,v,p) = (1, 0, 1000)   r < 0.4
(ρ,v,p) = (1, 0, 1)   r > 0.4
Points refer to the main diagonal of the 3D cartesian box.
Solid lines are from a spherical 1D simulation (N = 800)
Even at low resolution the main features are resolved.
Here the highest Lorentz factor is > 25



Astrophysical Jet 2–D Cylindrical Symmetry:
Inlet radius   r < 1
Inflow jet speed = 0.99c , Relativistic Mach = 17.9 , Density ratio η = 1/100
Plot at times T = 20, 30, 40   Logaritmic values of density are shown.
Despite the use of a two wave approximate solver, Kelvin–Helmholtz instabilitie are nicely defined as well as the external bow–shock, the internal Mach disk, and other shocks reflected off the axis.