



# Universal density profiles with a central cusp

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**Abstract.** Grounding on both theoretical predictions and observational evidences, gas and stars contribute to about one tenth of the total mass of galaxies, and are lying within a more extended subsystem made of some (still unspecified) dark matter. The knowledge of the density profile of such dark matter halos is important, in that it influences the contraction of the baryonic subsystem, and then the structure of the virialized galaxy. In the current attempt, the general theory of realistic density profiles is performed, and explicit expressions of some physical parameters are derived. We are interested in triaxial, ellipsoidal structures where the isodensity surfaces are similar and co-axial ellipsoids.

**Key words.** cosmology: dark matter - galaxies: halos

## 1. Introduction

Grounding on both theoretical predictions and observational evidences, galactic gas and stars are lying within a more extended subsystem made of some (still unspecified) dark matter. The knowledge of the density profile of such dark matter halos is important, in that it influences the contraction of the baryonic subsystem, and then the structure of the virialized galaxy. In performing the general theory of realistic density profiles we take into account the following family of density profiles (e. g. Zhao 1996):

$$\rho\left(\frac{r}{r^\dagger}\right) = \frac{\rho^\dagger}{(r/r^\dagger)^\gamma [1 + (r/r^\dagger)^\alpha]^\chi} \quad (1)$$

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$$\chi = \frac{\beta - \gamma}{\alpha} ; \quad (2)$$

for a suitable choice of exponents,  $\alpha$ ,  $\beta$ ,  $\gamma$ . This family includes both cuspy profiles first proposed by Navarro et al. (1995, 1996, 1997),  $(\alpha, \beta, \gamma) = (1, 3, 1)$ , and the so called modified isothermal profile,  $(\alpha, \beta, \gamma) = (2, 2, 0)$ , which is the most widely used model for the halo density distribution in analyses of observed rotation curves. It also includes the perfect ellipsoid e. g. de Zeeuw (1985),  $(\alpha, \beta, \gamma) = (2, 4, 0)$ , which is the sole (known) ellipsoidal density profile where a test particle admits three global integrals of motion. Finally, it includes the Hernquist (1990) density profile,  $(\alpha, \beta, \gamma) = (1, 4, 1)$ , which closely approximates the de Vaucouleurs  $r^{1/4}$  law for elliptical galaxies. In dealing with the formation of dark matter halos from hierarchical clustering in both CDM and  $\Lambda$ CDM scenarios, recent high-resolution simulations allow  $(\alpha, \beta, \gamma) = (3/2, 3, 3/2)$ , as a best fit

(Ghigna et al. 2000; Fukushige & Makino 2001) as first advocated by Moore et al. (1998, 1999)

## 2. The model

We take into account triaxial, ellipsoidal structures. The isopycnic surfaces are defined by the following law:

$$\rho = \rho^\dagger f(\xi) \ ; \ f(1) = 1 \ ; \quad (3)$$

$$\xi^2 = \sum_{\ell=1}^3 \frac{x_\ell^2}{(a_\ell^\dagger)^2} \ ; \ 0 \leq \xi \leq \Xi \ ; \quad (4)$$

where  $\rho^\dagger = \rho(1)$ ,  $a_\ell^\dagger$ , are the density and the semiaxes, respectively, of a reference isopycnic surface, and  $\Xi$  corresponds to the truncation isopycnic surface, related to semiaxes  $a_\ell$ . We suppose that the density profile is (i) self-similar, in the sense that it has the same expression, independent of time (e.g., Fukushige & Makino 2001) and (ii) universal, in the sense that it has the same expression, independent of halo mass, initial density perturbation spectrum, or value of cosmological parameters (e.g., Navarro et al. 1997; Fukushige & Makino 2001). Though Eq. 1 implies null density at infinite radius, the mass distribution has necessarily to be truncated for two types of reasons. First, the presence of neighbouring systems makes the tidal radius an upper limit. Second, the total mass, deduced from Eq. 1 for an infinitely extended configuration, is divergent, at least with regard to the special choices of exponents,  $(\alpha, \beta, \gamma) = (1, 3, 1)$ , and  $(\alpha, \beta, \gamma) = (3/2, 3, 3/2)$ . The region enclosed within the truncation boundary has to be intended as representative of the quasi static halo interior, leaving aside the surrounding material which is still infalling. It is worth remembering that the total mass can be finite even for an infinitely extended configuration, provided the related density profile is sufficiently steep e.g.,  $(\alpha, \beta, \gamma) = (1, 4, 1)$ .

## 3. Conclusions

In the following table we report the main functions and profile parameters related to our model. The profile parameters depend on a single unknown, i.e. the truncated, scaled radius,  $\Xi$ . The profile parameter,  $\nu_J$ , is related to the special case of constant rotational velocity on the equatorial plane, with regard to spheroidal configurations. Rigidly rotating, ellipsoidal configurations correspond to  $\nu_J = \nu_I$ .

function	definition
$f(\xi)$	$\frac{\rho(\xi)}{\rho_0}$
$P(\xi)$	$2 \int_\xi^{\rho_0} f(\xi) \xi d\xi$
$F(\xi)$	$2 \int_\xi^\Xi f(\xi) \xi d\xi$
$\nu_M$	$\frac{M}{M_0}$
$\nu_{\bar{\rho}}$	$\frac{\bar{\rho}}{\rho_0}$
$\frac{\nu_{eq}}{[\eta(\Xi)/\eta_0]^{1/2}}$	$\frac{\nu_{eq}(\Xi)}{(v_0)_{eq}} \left[ \frac{\eta_0}{\eta(\Xi)} \right]^{1/2}$
$\nu_I$	$\frac{I_{pp}}{M a_p^2}$
$\nu_{sel}$	$\frac{-(E_{sel})_{pp} a_1}{GM^2 \epsilon_{p2} \epsilon_{p3} A_p}$
$\nu_J$	$\frac{1}{\nu_M \Xi} \int_0^\Xi f(\xi) \xi^3 d\xi$

Further details will be published elsewhere (Caimmi & Marmo 2003).

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