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A simple and efficient high-resolution shock-capturing method for relativistic fluid dynamics

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Abstract. Relativistic flows play an essential role in high energy astrophysics, and relativistic shocks are supposed to be the place where particle acceleration occurs. Among the astrophysical sources of such flows and shocks there are AGNs, and the related jets, the GRB fireball, and pulsar wind nebulae. We present an easy to implement multidimensional shock-capturing relativistic hydrodynamics (RHD) scheme. RHD numerical schemes are generally more expensive than their corresponding Eulerian version, essentially for the complexity of the relation between conservative and primitive variables. Our scheme is based on third order CENO reconstruction and on averaged Riemann solvers which do not require characteristic decomposition. The scheme is efficient and robust, even in multidimensional simulations, and can cope with very high Lorentz factors, giving results comparable with those of more sophisticated methods.

Key words. hydrodynamics – relativity – shock waves – methods: numerical

1. Introduction

In the last decade, high resolution shock-capturing methods of Godunov type, successfully applied in classical fluid dynamics, have started to be employed for the case of relativistic hydrodynamics as well (Balsara 1994; Donat et al. 1998; Aloy et al. 1999). All these schemes solve a conservative form of the discretized equation in order to capture weak solutions and satisfy jump condition, usually with a reconstruction phase to achieve second order resolution. Here an efficient and easy to implement RHD shock-capturing scheme is presented, based on the algorithm developed by Londrillo & Del Zanna (2000). The scheme is based on third order Convex Essentially Non-Oscillatory (CENO) (Liu & Osher 1998) finite difference interpolation routines and on central type averaged Riemann solvers which do not make use of time-consuming characteristic decomposition, having in mind the extension to the magnetohydrodynamic regime (Del Zanna et al. 2003).

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2. The numerical code

The equations of special relativistic hydrodynamics may be cast (as in the Eulerian case) in the 3D conservative form $\partial \mathbf{u}/\partial t = \sum_{i=1}^{3} \partial f(\mathbf{u})^i/\partial x^i$, where the conserved variables are $\mathbf{u} = [\rho\gamma, w\gamma^2 v^j, w\gamma^2 - p]^T$, and the relative fluxes are $f(\mathbf{u})^i = [\rho\gamma v^i, w\gamma^2 v^j v^i + p\delta^{ij}, w\gamma^2 v^i]^T$, with $\gamma = (1-v^2)^{-1/2}$, where we have assumed an ideal gas equation of state ($w = \rho + \Gamma p/(\Gamma - 1)$). The main features of our code are: (1) Point values rather than cell averages are used (finite differences instead of finite volumes), making the extension from 1D to 3D easier. (2) The semi-discrete form of equations is solved, so that time integration can be achieved with any solver for ordinary differential equations. (3) Neither characteristic decomposition nor accurate Riemann solver are required: fluxes are derived component-wise, thus achieving simplicity and efficiency. (4) Primitive variables are recovered by solving a single equation for the Lorentz factor iteratively. (5) Good computing efficiency: all simulations have been run on a 1 GHz PC. The code is also fully parallelized with MPI and has been tested on various supercomputers.

The time integration is performed with a TVD third order Runge-Kutta method. For every subcycle and for every direction the following steps are taken: (1) Central values of primitive variables are recovered from conservative ones in every cell. (2) Primitive variables are reconstructed at cell interfaces to give a left (L) and right (R) state. The interpolation is done separately on each variable with CENO third order technique, using min-mod (MM) or monotonized-centered (MC) limiters near discontinuities. (3) At each inter-cell point fluxes are computed with approximate Riemann solvers $f^i_{HLL} = \frac{\alpha^+ f^i_L + \alpha^- f^i_R - \alpha^+\alpha^-(u^i_R - u^i_L)}{\alpha^+ + \alpha^-}$ or the Lax-Friedrichs solver (same as f_{HLL} but with $\alpha^+ = \alpha^-$) where $\alpha^\pm = \max(0, \pm\lambda^\pm(\mathbf{u}_L), \pm\lambda^\pm(\mathbf{u}_R))$ and $\lambda^\pm = \frac{v_\parallel(1-c_s^2) \pm \sqrt{(1-v^2)(1-v_\parallel^2 - v_\perp^2 c_s^2)}}{1-v^2 c_s^2}$ where the parallel and perpendicular suffixes refer to the spatial direction of integration. (4) Flux derivatives are reconstructed at cell-centers using another CENO routine to the same accuracy order.

3. Conclusions

Compared with other schemes proposed for relativistic astrophysical problems over the last decade in the literature, our method is extremely simple and efficient since neither eigenvector decomposition nor Riemann solvers are involved. The algorithm is able to compensate for the large smearing of contact discontinuities due to the use of solvers based on just one or two characteristic speeds, and above all to the reconstruction method which is the same for all quantities, so that too steep limiters cannot be used. The position of shocks and rarefaction waves is well defined and the code is able to resolve both turbulent fields and discontinuities appearing together. Spherical and cylindrical geometries as well as different boundary conditions have also been tested, and the scheme has proved to be stable up to high (~ 200) Lorentz factor. For a more detailed explanation of the code as well as the reconstruction procedure see Del Zanna & Bucciantini (2002).

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