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Hexapod kinematics for secondary mirror aberration control

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Abstract. This work deals with active correction of the aberrations in a telescope by moving the secondary mirror. A special attention is dedicated to the case of a secondary mirror whose motions are controlled by a 6-6 Stewart Platform (generally called by astronomers simply "hexapod", even if this term is more general). The kinematics of the device is studied. The non trivial forward kinematics problem is solved by an iterative algorithm fitting the necessities of an active optics system and fast enough to be used in a closed loop feedback control.

Key words. Active Optics - Kinematics - Stewart platform - Software

1. Active optics by M2: fundamentals

In a two-mirror telescope equipped with an active optics system usually M2 is used to correct the aberrations originated by optical reasons, i.e. defocus and coma. The tilt correction is usually not performed with M2 because the tilt effect is a pointing error which just moves the image in the focal plane, and can be actively compensated by the autoguider. Zernike polynomials are commonly used to describe the "optical" aberrations; on the contrary the elastic modes can be the best choice to describe the errors whose origin is mechanical rather then optical, which are corrected by changing the M1 shape. Both Zernike and elastic modes are sets of mutually orthogonal functions, mixing them together in the aberrations fitting procedure there is a small but negligible non orthogonality. In order to set-up a correction system of the aberrations by moving M2 it is necessary to derive relationships between motions and generated aberrations. The polynomials used to represent defocus and coma aberrations are reported in tab.1 (they are not the standard Zernike polynomials but the so-called quasi-Zernike ones). The aberrations produced by misalignments in a two mirrors reflecting telescope are analyzed in details in Schroeder (1987), Wilson (1996) and Wilson (1999).

2. Corrections using an hexapod

If an hexapod is used to move the secondary mirror, the basic equations of the device must be known. In the following the "hexapod" is supposed to be a particular implementation of a 6-6 Stewart platform composed by a fixed platform and a mobile one. In both platforms the leg joints form a regular hexagon where all points belong to the same plane and lie on a circle (Fig.1).

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Table 1. Quasi-Zernike polynomials

Sym	Order	Polynomial	Optical interpr.
0	2	r^2	Defocus
1	2	$r^3 \cos\theta$	Coma

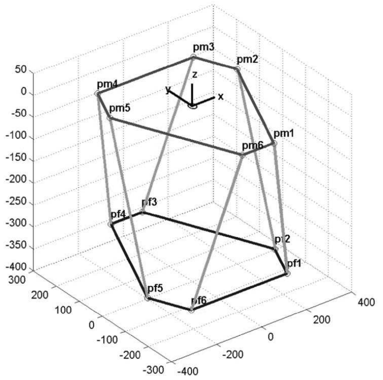

Fig. 1. Hexapod 3d coordinate system; z-axis points along the positive optical direction (towards M1)

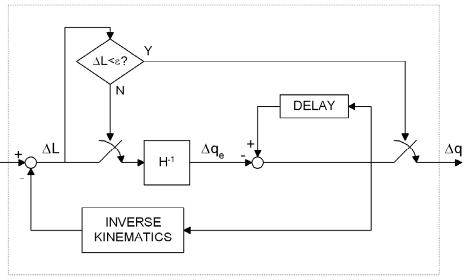

Fig. 2. Forward kinematics iterative algorithm

2.1. Inverse kinematics

The "Inverse Kinematics Problem" is: known the position of the mobile points (i.e. the position and orientation of the mobile platform) and of course of the fixed ones, find the length of the legs. The coordinates of the mobile points after a generic movement expressed through a rotational and a traslational part are (i=1...6):

$$P'_{m_i} = R_{rpy} P_{m_i} + T$$

The lengths of the legs after the roto traslation are:

$$L_i = \sqrt{\Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2}$$

2.2. Forward kinematics

The "Forward Kinematics Problem" is: known the lengths of the legs, find the position of the mobile points (i.e. position and orientation of the mobile platform). For a general 6-6 Stewart platform there is no simple analytical solution to the forward kinematics problem.

Nevertheless for some particular kinds of 6-6 Stewart platforms a considerable help may come from geometric consideration and the problem can be solved in closed form coming to a set of possible solutions, the only residual problem being the necessity to select the right one (but sometimes some solutions of the set can be automatically discarded for physical constraints). Many different approaches exist in literature (e.g. Didrit et al.(1998), Husty (1996), Ji et al.(2001), Lee et al. (2001), Yang et al. (1998), Zhang et al. (1991)), especially aiming to speed-up the computation for real time control.

But in the active optics case the corrections cannot be applied at high frequency because of the slow integration time needed to have a good measure of the optical system aberrations (the exposure time of the image analyser is usually not less than 30 secs to integrate out any aberration effect not depending on the telescope). Also, the range of motions of an hexapod used to compensate just defocus and coma is very narrow.

The algorithmic approach discussed in the following is well suited to the usage of an hexapod system in active optics applications. The problem formulation is: how to find $q = \begin{pmatrix} x & y & z & \psi & \vartheta & \varphi \end{pmatrix}$ knowing Ł$_i$ (i=1...6). Following the Newton's method the non linear equation of the inverse kinematics can be linearized around a generic initial point q_0 (not necessarily coincident with the rest position of the platform).

$$L_i = L_{i0} + \left.\frac{dG}{dq}\right|_{q=q_0} (q - q_0) + ...$$

$$\Delta q \approx \left(\left.\frac{dG}{dq}\right|_{q=q_0} \right)^{-1} \Delta L_i = H^{-1} \Delta L_i$$

Once computed the terms of matrix H, the forward kinematics algorithm can be started. The input is the vector of the the lengths of the legs Ł$_i$ (i=1...6), the output is the vector q describing the position and orientation of the mobile platform (or equivalently the transformation matrix R(q)). The algorithm (Fig.2) is the following:

1) Start from an initial guess $\hat{\Delta q}$

2) Apply inverse kinematics, i.e. compute for i=1...6:

$$\hat{P}_{m_i} = R\left(\hat{\Delta q}\right) P_{m_i}$$

$$\Rightarrow \hat{L}_i = \sqrt{\left(\hat{P}_{m_i} - P_{f_i}\right)^T \left(\hat{P}_{m_i} - P_{f_i}\right)}$$

3) Compute the error between the real lengths of the legs and their estimated value:

$$\Delta L_i = \hat{L}_i - L_i \Rightarrow \Delta L = \begin{pmatrix} \Delta L_1 \\ \vdots \\ \Delta L_6 \end{pmatrix}$$

If ΔL is sufficiently little (e.g. a simple criterium can be $\Delta L_i < \varepsilon$ $\forall i \in [1,6]$) than the algorithm can stop

4) Compute the displacement vector Δq_e due to ΔL:

$\Delta q_e = H^{-1} \Delta L$

5) Update the guess value:

$\hat{\Delta q} = \hat{\Delta q} - \Delta q_e$

6) Go to step 2 while $\Delta L_i > \varepsilon$

The drawback of this algorithm is that the number of iterations is not constant and a priori unpredictable, therefore it is not well suited to real-time control. Nevertheless as it has been discussed in active optics the rate of M2 corrections is sufficiently low to justify the use of this iterative method.

The forward kinematics iterative algorithm has proved to be fast enough in any possible condition, even for postures much "far" away from the initial point than the ones inside the very little range of motions of the hexapods used in active optics. The maximum time needed to perform the forward kinematics iterations is by far (some order of magnitudes) lower than the time between two M2 position corrections.

References

Didrit, O., et al. 1998, IEEE Trans. Robot. Automn, 14, 259

Husty, M. 1996, Mech. Mach. Theory, 31, 365

Ji, P., et al. 2001, IEEE Trans. Robot. Automn, 17, 522

Lee, T., Shim, J. 2001, Proc. of the 2001 IEEE Int. Conf. Robot. Automn, 1301

Schroeder, D.J. 1987, Astronomical Optics, Academic Press

Yang, J., Geng, Z.J. 1998, IEEE Trans. Robot. Automn, 14, 503

Wilson, R. 1996, Reflecting Telescope Optics I, Springer Verlag

Wilson, R. 1999, Reflecting Telescope Optics II, Springer Verlag

Zhang, C., Song, S. 1991, Proc. of the 1991 IEEE Int. Conf. Robot. Automn, 2676