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A new modeling approach for DACs and SACs regions in the atmospheres of hot emission stars

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Abstract. The presence of Discrete Absorption Components (DACs) or Satellite Absorption Components (SACs) is a very common phenomenon in the atmospheres of hot emission stars (Danezis et al. 2003; Lyratzi & Danezis 2004) and result to the complex line profiles of these stars. The shapes of these lines are interpreted by the existence of two or more independent layers of matter nearby a star. These structures are responsible for the formation of a series of satellite components for each spectral line. Here we will present a model reproducing the complex profile of the spectral lines of Oe and Be stars with DACs and SACs (Danezis et al. 2003; Lyratzi & Danezis 2004). In general, this model has a line function for the complex structure of the spectral lines with DACs or SACs and include a function L that considers the kinematic (geometry) of an independent region. In the calculation of the function L we have considered the rotational velocities of the independent regions, as well as the random velocities within them. This means that the new function of L is a synthesis of the rotational distribution and a physical Gaussian. Finally, we calculate the optical depth (ξ) and the column density (d) of each independent density region.

Key words. Stars: hot emission – Stars: modeling atmospheres – DACs

1. Introduction

One of the most important phenomena in the spectra of hot emission stars is the DACs (Discrete Absorption Components) phenomenon (Peton 1974; Underhill 1975; Lamers et al. 1982; Sahade et al. 1984; Sahade & Brandi 1985; Hutsemékers 1985; Danezis 1984, 1987; Danezis et al. 1991, 2003).

DACs are discrete but not unknown absorption spectral lines. They are spectral lines of the same ion and the same wavelength as a main spectral line, shifted at different $\Delta\lambda$, as they are created in different density regions which rotate and move radially with different velocities (Danezis et al. 2003; Lyratzi & Danezis 2004). DACs are lines, easily observed, in the spectra of Be stars of luminosity class III, because the regions that give rise to such lines rotate with low velocities and move radially with high velocities. However, if the regions that give rise to such lines rotate with large velocities and move radially

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with small velocities, the produced lines are much broadened and little shifted. As a result they are blended among themselves as well as with the main spectral line and thus they are not discrete. In such a case the name Discrete Absorption Component is inappropriate and we use only the name SACs (Satellite Absorption Components).

Danezis et al. (1991, 1998, 2000a,b,c, 2002a,b, 2003) and Laskarides et al. (1992a,b) proposed a new model to explain the complex structure of the density regions of hot stars, where the spectral lines that present SACs or DACs are created.

The main hypothesis of this model is that the atmospherical region where a specific line is created is not continuous, but it is composed of a number of successive independent absorbing density regions, a number of emission regions and an external general absorption region.

2. Description of the model

2.1. The line profile function

By solving the equations of radiation transfer through a complex structure as the one described, we conclude to a function for the line's profile, able to give the best fit for the main spectral line and its Satellite Components in the same time.

The line profile function is the following:

$$I_\lambda = [I_{\lambda 0}\Pi + \Sigma]e^{-x_g} \tag{1}$$

where: $I_{\lambda 0}$: is the initial radiation intensity, $\Pi = \prod_i e^{-x_{ai}}$ and $\Sigma = \sum_j S_{\lambda ej}(1 - e^{-x_{ej}})$, $e^{-x_{ai}}$, $e^{-x_{ei}}$, e^{-x_g}: are the distribution functions of the absorption, emission an general absorption lines respectively.

This function I_λ does not depend on the geometry of the regions which create the observed feature.

2.2. The rotation model

One of the main hypotheses when we constructed the old version of the model (rotation model) was that the line's width $\Delta\lambda$ is only a rotational effect and we consider spherical symmetry for the independent density regions, which create the satellite components. This means that the random velocities were very low and they did not contribute to the line's broadening. In such a case the function (1) becomes:

$$I_\lambda = [I_{\lambda 0}\Pi_R + \Sigma_R]e^{-L_g\xi_g} \tag{2}$$

where: $I_{\lambda 0}$: is the initial radiation intensity, $\Pi_R = \prod_i e^{-L_i\xi_i}$ and $\Sigma_R = \sum_j S_{\lambda ej}\left(1 - e^{-L_{ej}\xi_{ej}}\right)$, L_i, L_{ej}, L_g: are the distribution functions of the absorption coefficients $k_\lambda i$, $k_\lambda ej$, $k_\lambda g$ respectively. ξ is the optical depth, $S_\lambda ej$: is the source function, which, at the moment when the spectrum is taken, is constant.

In this case we can calculate that L is the following function:

$$L(\lambda) = \int_{-\theta_0}^{\theta_0} R\cos\theta[P_1(\lambda) - P_2(\lambda)]d\theta$$

where R is the radius of the spherical density region,
$P_1(\lambda) = \frac{\arctan[\lambda - \lambda_{lab}(1 - z_0\cos\theta)]}{\lambda_{lab}z_0\cos\theta}$ and $P_2(\lambda) = \frac{\arctan[\lambda - \lambda_{lab}(1 + z_0\cos\theta)]}{\lambda_{lab}z_0\cos\theta}$
where λ_{lab} is the laboratory wavelength of the spectral line produced by a particular ion and

$$z_0 = \frac{\Delta\lambda_{rotation}}{\lambda_{lab}} = \frac{V_{rotation}}{c}$$

where $\Delta\lambda_{rotation}$ is the width and $V_{rotation}$ the apparent rotational velocity of the i density shell of matter.

The polynomial $P(\lambda) = P_1(\lambda) - P_2(\lambda)$ can be well approximated by the polynomial $f(\lambda)$, where

$$f(\lambda) = \begin{cases} 1, if|\lambda - \lambda_0| < \rho \\ 0, if|\lambda - \lambda_0| \geq \rho \end{cases}, \text{ where}$$

$$\rho = \frac{\lambda_0 z_0\cos\theta}{1 - z_0^2\cos^2\theta}$$

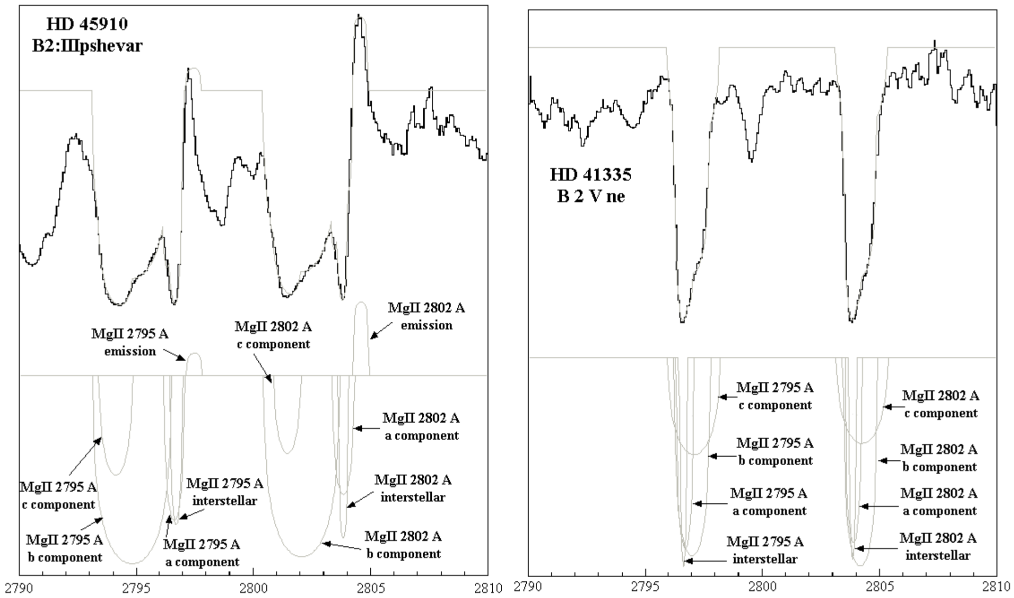

Fig. 1. The best fit is not just the graphical composition of some distribution functions. The reproduced feature is the result of the final function of the model. The Mg II line profile of the star HD 45910, which presents DACs and the star HD 41335, which presents SACs are produced in the same way. The only difference between them is that the components of HD 41335 are much less shifted and thus they are blended among themselves. The black line presents the observed spectral line's profile and the gray one the model's fit. The differences between the observed spectrum and its fit are some times hard to see, as we have accomplished the best fit. We also present all the components which contribute to the observed features, separately.

λ_0 is the observed wavelength of the center of the spectral line, $\lambda_0 = \lambda_{lab} + \Delta\lambda_{rad}$, and $\Delta\lambda_{rad}$ is the radial Doppler shift:

$$\frac{\Delta\lambda_{rad}}{\lambda_{lab}} = \frac{V_{rad}}{c}$$

Thus, for

$$|\lambda - \lambda_0| < \rho = \frac{\lambda_0 z_0 \cos\theta_0}{1 - z_0^2 \cos^2\theta_0}$$

(where the values of λ are taken in the wavelength range we want to reproduce) and if $\theta \leq \theta_0$, then $P(\lambda) = 1$ and the distribution $L(\lambda)$ yields:

$$L(\lambda) \cong \int_{-\theta_0}^{\theta_0} R\cos\theta d\theta.$$

We normalize for $R = 1$ and we have:

$L(\lambda) \cong \int_{-\theta_0}^{\theta_0} \cos\theta d\theta = [\sin\theta]_{-\theta_0}^{\theta_0} = 2\sin\theta_0$. We normalize and we have:

$L(\lambda) \cong \sin\theta_0 = \sqrt{1 - \cos^2\theta_0}$

By solving the equation

$$\Delta\lambda_{rotation} = \frac{\lambda_0 z_0 \cos\theta_0}{1 - z_0^2 \cos^2\theta_0}$$

we have

$$\cos\theta_0 = \frac{-\lambda_0 \pm \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation} z_0}$$

As θ_0 lies between $-\pi/2$ and $\pi/2$ and $cos\theta_0 \geq 0$ we have

$$\cos\theta_0 = \frac{-\lambda_0 + \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation} z_0}$$

If

$$\cos\theta_0 = \frac{-\lambda_0 + \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation} z_0} < 1$$

then $L(\lambda) = \sqrt{1-\cos^2\theta_0}$ and
if

$$\cos\theta_0 = \frac{-\lambda_0 + \sqrt{\lambda_0^2 + 4\Delta\lambda_{rotation}^2}}{2\Delta\lambda_{rotation} z_0} \geq 1$$

then $L(\lambda) = 0$.

The spectral line's profile, which is formed by the i density shell of matter, must be accurately reproduced by the function $e^{-L_i\xi_i}$ by applying the appropriate values of V_{rot_i}, V_{rad_i} and ξ_i.

Using the best model's fit for a complex spectral line, we can calculate the apparent radial velocity (V_{rad_i}), the apparent rotational velocity (V_{rot_i}) and the optical depth (ξ_i) of the region in which the main spectral line and its SACs are created.

3. The new modeling approach

In the present work we propose a new approach of the problem, as we also consider the parameter of random velocities in the calculation of the distribution function L. This new L is a synthesis of the rotational distribution L_r that we had presented in the old rotational model and a Gaussian. This means that the new L has two limits, the first one give us a Gaussian and the other the old rotational L_r.

3.1. The new calculation of the distribution functions L

Let us consider a spherical shell and a point A_i in its equator.

If the laboratory wavelength of a spectral line that arises from A_i is λ_{lab}, the observed wavelength will be $\lambda_0 = \lambda_{lab} + \Delta\lambda_{rad}$

If the spherical density region rotates, we will observe a displacement $\Delta\lambda_{rot}$ and the new wavelength of the center of the line λ_i is:
$\lambda_i = \lambda_0 \pm \Delta\lambda_{rot}$ and $\Delta\lambda_{rot} = \lambda_0 z \sin\varphi$, $z = \frac{V_{rot}}{c}$, V_{rot} is the rotational velocity of the point A_i. This means that

$\lambda_i = \lambda_0 \pm \lambda_0 z\sin\varphi = \lambda_0(1 \pm z\sin\varphi)$ and if $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$ then $\lambda_i = \lambda_0(1 - z\sin\varphi)$

If we consider that the spectral line profile is a Gaussian distribution we have:

$$P(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{\lambda-\kappa}{\sigma\sqrt{2}}\right]^2} \tag{3}$$

where κ is the mean value of the distribution and in the case of the line profile it indicates the center of the spectral line that arises from A_i. This means that:

$$P(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{\lambda-\lambda_0(1-z\sin\varphi)}{\sigma\sqrt{2}}\right]^2} =$$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[\lambda-\lambda_0(1-z\sin\varphi)]^2}{2\sigma^2}}$$

The distribution function for all the semi-equator is:

$$L(\lambda) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[\lambda-\lambda_0(1-z\sin\varphi)]^2}{2\sigma^2}} \cos\varphi d\varphi$$

If we set $\sin\varphi = x$ and $u = \frac{\lambda-\lambda_0(1-zx)}{\sqrt{2}\sigma}$, then

$$L(\lambda) = \frac{1}{\lambda_0 z\sqrt{\pi}} \int_{\frac{\lambda-\lambda_0(1+zx)}{\sqrt{2}\sigma}}^{\frac{\lambda-\lambda_0(1-zx)}{\sqrt{2}\sigma}} e^{-u^2} du$$

$$L(\lambda) = \frac{1}{\lambda_0 z\sqrt{\pi}}(Q_1 - Q_2)$$

$$Q_1 = \int_0^{\frac{\lambda-\lambda_0(1-zx)}{\sqrt{2}\sigma}} e^{-u^2} du \text{ and}$$

$$Q_2 = \int_0^{\frac{\lambda-\lambda_0(1+zx)}{\sqrt{2}\sigma}} e^{-u^2} du$$

These two integrals have the form of a known integral erf(x): $erf(x) = \frac{2}{\pi}\int_0^x e^{-u^2} du$

So:

$$L(\lambda) = \frac{1}{2\lambda_0 z}[erf_1 - erf_2] \tag{4}$$

where:

$$erf_1 = erf\left(\frac{\lambda - \lambda_0(1 - zx)}{\sqrt{2}\sigma}\right)$$

and

$$erf_2 = erf\left(\frac{\lambda - \lambda_0(1 + zx)}{\sqrt{2}\sigma}\right)$$

The distribution function from the semi-spherical region is:

$$L_f(\lambda) = \frac{1}{2\lambda_0 z}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[erf_{f1} - erf_{f2}\right]\cos\theta d\theta \qquad (5)$$

where:

$$erf_{f1} = erf\left(\frac{\lambda - \lambda_0}{\sqrt{2}\sigma} + \frac{\lambda_0 z}{\sqrt{2}\sigma}\cos\theta\right)$$

and

$$erf_{f2} = erf\left(\frac{\lambda - \lambda_0}{\sqrt{2}\sigma} - \frac{\lambda_0 z}{\sqrt{2}\sigma}\cos\theta\right)$$

(Method Simpson)

This $L_{final}(\lambda)$ is the distribution that replaces the rotation distribution L that Danezis et al. (2003) proposed.

4. Discussion

In the proposed distribution an important factor is $m = \frac{\lambda_0 z}{\sqrt{2}\sigma}$. This factor indicates the kind of the distribution that can fit the line profile.

1. If $m \cong 3$ we have a mixed distribution. The line broadening is an effect of two equal reasons: (a) the rotational velocity of the spherical region and (b) the random velocities of the ions.
2. If $m \cong 500$ the line broadening is only an effect of the rotational velocity and the random velocities are very low. In this case the profile of the line is the same with the profile that we can produce using the Danezis et al. (2003) rotation model.
3. If $m < 1$ the line broadening is only an effect of random velocities and the line distribution is Gaussian.

4.1. The column density

An important point of our model is the calculation of the column density. Lets start from the definition of the optical depth: $\tau = \int_0^s k\rho ds$, where τ is the optical depth (no units), k is the absorption coefficient ($\frac{cm^2}{gr}$), ρ is the density of the absorbing region ($\frac{gr}{cm^3}$) and s is the geometrical depth (cm).

In the model we set $k = L\Omega$, so $\tau = \int_0^s L\Omega\rho ds$, where L is the distribution function of the absorption coefficient k and has no units, Ω equals 1 and has the units of k ($\Omega = 1\frac{cm^2}{gr}$).

We consider that for the moment of the observation and for a significant ion, k has a significant, constant value, so k (and thus L and Ω) may come out of the integral. So: $\tau = L\int_0^s \Omega\rho ds$. We set $\xi = \int_0^s \Omega\rho ds$ and τ becomes $\tau = L\xi$

4.1.1. Absorption lines

About every one of ξ (henceforth called ξ_i), we have:

$$\xi_i = \int_0^s \Omega\rho ds \Rightarrow \xi_i = \Omega\int_0^s \rho ds \Rightarrow$$

$$\frac{\xi_i}{\Omega} = \int_0^s \rho ds$$

We set $\sigma_i = \frac{\xi_i}{\Omega} = \int_0^s \rho ds$. As $\Omega = 1\frac{cm^2}{gr}$, it contributes only to the units and σ_i takes the value of ξ_i.

For each λ_i along the spectral line, we extract a σ_i from each ξ_i. The program we use calculates the ξ_i for the center of the line. This means that from this ξ_i we can measure the respective σ_i.

If we add the values of all σ_i along the spectral line then we have $\sigma = \sum_i \sigma_i$ (in $\frac{gr}{cm^2}$), which is the surface density of the absorbing matter, which creates the spectral line.

If we divide σ with the atomic weight of the ion which creates the spectral line, we extract the number density of the absorbers, meaning the number of the absorbers per square centimeter ($n = \frac{\sigma}{AW}$ (in cm^{-2})).

This number density corresponds to the energy density which is absorbed by the whole matter which creates the observed spectral line ($\frac{E}{AW}$ (in $\frac{erg}{cm^2}$)) and which is calculated by the model.

It is well known, that each absorber absorbs the specific amount of the energy needed for the transition which creates the specific line. This means that if we divide the calculated energy density ($\frac{E}{AW}$) with the energy needed for the transition, we obtain the column density (in cm^{-2}).

4.1.2. Emission lines

In the case of the emission lines we have to take into account not only ξ_e, but also the source function S, as both of these parameters contribute to the height of the emission lines. So in this case we have:

$$S\xi_e = \frac{j}{k}\int_0^s \Omega\rho_e ds$$

where: j is the emission coefficient ($\frac{erg}{gr\cdot s\cdot rad\cdot A}$), k is the absorption coefficient ($\frac{cm^2}{gr}$), ρ_e is the density of the emitting region ($\frac{gr}{cm^3}$) and s is the geometrical depth (cm).

We set $k = L\Omega$ where L is the distribution function of the absorption coefficient k and has no units, Ω equals 1 and has the units of k ($\Omega = 1\frac{cm^2}{gr}$) and $j = L_e\Omega_e$, where L_e is the distribution function of the emission coefficient j and has no units and Ω_e equals 1 and has the units of j ($\Omega_e = 1\frac{erg}{gr\cdot s\cdot rad\cdot \text{Å}}$).

As we did in the case of the absorption lines, we may consider that Ω may come out of the integral. So:

$$S\xi_e = \frac{j}{k}\int_0^s \Omega\rho_e ds = \frac{L_e\Omega_e}{L\Omega}\int_0^s \Omega\rho_e ds =$$

$$= \frac{L_e\Omega_e}{L\Omega}\Omega\int_0^s \rho_e ds = \frac{L_e\Omega_e}{L}\int_0^s \rho_e ds$$

As in the model we use the same distribution for the absorption and for the emission (rotation distribution) $L_e = L$. So:

$$S\xi_e = \Omega_e\int_0^s \rho_e ds \Rightarrow \frac{S\xi_e}{\Omega_e} = \int_0^s \rho_e ds$$

We set $\sigma_e = \frac{S\xi_e}{\Omega_e} = \int_0^s \rho_e ds$.

As $\Omega_e = 1\frac{erg}{gr\cdot s\cdot rad\cdot \text{Å}}$, it contributes only to the units and σ_e takes the value of $S\xi_e$.

For each λ_i along the spectral line, we extract a σ_i from each $S\xi_e$. The program we use calculates the ξ_e for the center of the line and the S. This means that from this ξ_e and S we can measure the respective σ_i.

If we add the values of all σ_i along the spectral line, we have $\sigma = \sum_i \sigma_i$ (in $\frac{gr}{cm^2}$), which is the surface density of the emitting matter, which creates the spectral line. If we divide σ with the atomic weight of the ion which creates the spectral line, we extract the number density of the emitters, meaning the number of the emitters per square centimeter ($n = \frac{\sigma}{AW}$ (in cm^{-2})). This number density corresponds to the energy density which is emitted by the whole matter which creates the observed spectral line ($\frac{E}{AW}$ (in $\frac{erg}{cm^2}$)) and which is calculated by the model. It is well known, that each emitter emits the specific amount of the energy needed for the transition which creates the specific line. This means that if we divide the calculated energy density ($\frac{E}{AW}$) with the energy needed for the transition, we obtain the column density (in cm^{-2}).

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(Astrophysical spectroscopy of extragalactic objects).

References

Danezis, E. 1984, The nature of Be stars, PhD Thesis, University of Athens

Danezis, E. 1987, in Proc. IAU Colloq. No 92, Physics of Be Stars, eds. A. Slettebak and T. P. Snow (Cambridge University Press), p.149

Danezis, E., Theodossiou, E., & Laskarides, P. G. 1991, Ap&SS, 179, 111

Danezis, E., et al. 1998, Satellite spectral components and moving shells in the atmospheres of Early Type Stars - The example of HD 175754, in Proc. 65th Czech and 7th European Joint Conf., JENAM '98, Prague, Czech Republic

Danezis, E., et al. 2000a, A simple model for the complex structure of moving atmospheric shells in Oe and Be stars. The example of HD 36861, some first conclusions. JENAM 2000, May 29-June 3, Moscow, Russia

Danezis, E., et al. 2000b, A simple model for the complex structure of moving atmospheric shells in Oe and Be stars. The example of HD 66811, some first conclusions. JENAM 2000, May 29-June 3, Moscow, Russia

Danezis, E., et al. 2000c, A simple model for the complex structure of moving atmospheric shells in Oe and Be stars. The example of HD 152408, some first conclusions. JENAM 2000, May 29-June 3, Moscow, Russia

Danezis, E., et al. 2002a, A new model of density layers of matter in the expanding gaseous envelope of Oe and Be stars, in Proc. IAU Symp. 210, Modeling of stellar atmospheres, June 17 21, 2002, Uppsala, Sweden

Danezis, E., et al. 2002b, Applications of a new model of density layers of matter in the expanding gaseous envelope of Oe and Be stars, in Proc. IAU Symp. 210, Modeling of stellar atmospheres, June 17 21, 2002, Uppsala, Sweden

Danezis, E., et al. 2003, Ap&SS, 284, 1119.

Hutsemékers, D. 1985, A&AS, 60, 373

Lamers, H. J. G. L. M., Gathier, R., & Snow, T. P. 1982, ApJ, 258, 186

Laskarides, P. G., Danezis, E. & Theodossiou, E. 1992a, Ap&SS, 179, 13

Laskarides, P. G., Danezis, E., & Theodossiou, E. 1992b, Ap&SS, 183, 67

Lyratzi, E., & Danezis, E. 2004, in AIP Conference Proceedings 740, The Physics of Ionized Gases: 22nd Summer School and International Symposium on the Physics of Ionized Gases, Bajina Bašta, 23-27 August 2004, eds. Lj. Hadzijevski, T. Grozdanov and N. Bibić, pp. 458-473

Peton, A. 1974, Space Sci. Rev., 30, 481

Sahade, J., Brandi, E., & Fontela, J.M. 1984, A&AS, 56, 17

Sahade, J., & Brandi, E. 1985, RMxAA, 10, 229

Underhill, A. B. 1975, ApJ, 199, 691