



# Planetary formation and orbital stability in the $\gamma$ Cephei system

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**Abstract.** We numerically investigated under which conditions the giant planet detected in the  $\gamma$  Cephei system could form in the core accretion scenario despite the presence of the very close companion star. In our study we modeled the accretion of large planetary embryos by using of a N-body symplectic mapping code adapted to S type binary star system. We also studied the dynamical stability of the giant planet orbit by using the FMA method of Laskar, testing its reliability in such a scenario. Our results show that, under the assumption of a massive protoplanetary disk, the core accretion model can explain the formation of the planet. The final semimajor axis results quite sensitive to the initial density profile assumed for the dust component of the disk.

**Key words.** planetary systems: formation, stability, core accretion; stars: binary systems; numerical: symplectic mapping

## 1. Introduction

Among the presently known extrasolar planetary systems, 15 are in binary star systems and  $\gamma$  Cephei is the one having the closest companion star. According to Hatzes et al. (2003), the secondary star has an orbit with a semimajor axis of  $18.5 \pm 1.1$  AU and an eccentricity of  $0.361 \pm 0.023$ . The planet detected around the primary star, a K0 III giant star, has a mass of  $M \sin i = 1.7 \pm 0.4$  Jupiter masses and an orbital semimajor axis of 2.13 AU. Giant planets orbiting one of the stars in a binary system offer the possibility to test the core-accretion model for giant planet formation (Pollack et al., 1996) in a complex dynamical environment. The vicinity of a companion star on such

a highly eccentric orbit may prevent the formation of a giant planet in the frame of the core-accretion model because the companion star:

1. reduces the size of the accretion disk
2. excites high relative velocities between colliding bodies

Our work is intended to study the possibility of *in situ* formation of a giant planet in the  $\gamma$  Cephei system by modelling numerically the *giant impacts* stage, when low velocity collisions between large embryos form the core of the giant planet. This last stage precedes the one of rapid gas accretion which sets on after sufficient accumulation of mass in the core (about  $10M_{\oplus}$ ). Our study of this phase is based on computer dynamical simulations using a full N-body model: to compute the orbital evolution of the system we used Chambers'

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**Table 1.** : Summary of the results of the N-body simulations of the giant impacts phase.

Swarm Mass ( $M_{\oplus}$ )	Dust superficial density ( $g\ cm^{-2}$ )	Fraction of formed cores	Formation percent	Formation time scale
25	50	0/8	0%	> 10 Myrs
35	50	3/8	37.5%	8–10 Myrs
45	100	6/8	75%	4–6 Myrs
75	100	8/8	100%	1–2 Myrs

Mercury code (Chambers, 1999) and the **DPI** plug–in we wrote. In our simulations we were primarily interested on the timescale for core formation and on the distance scale problem, i.e. under which conditions a core can form at the observed distance of 2.1 AU from the primary star. The timescale problem is due to the observational evidence that circumstellar disks do not last longer than 10 Myr. Within this timescale, a core must have enough mass to attract the surrounding gas and to become a giant planet with a solid core.

To conclude our study of the planetary system in  $\gamma$  Cephei, we studied the orbital stability of  $\gamma$  Cephei b using the FMA method by Laskar et al. (1992) and we compared the results of our analysis with the ones present in literature, in order to test the reliability of this method when applied to binary star systems.

## 2. Numerical model and physical assumptions

Our simulations are intended to study the possibility of formation of a critical mass core before the dissipation of the gaseous component of the protoplanetary disk. We were not interested in simulating the gas infall and we just investigated the giant impacts phase by modeling the dynamical evolution of a swarm of protoplanetary embryos of lunar mass. In order to determine how the planetary formation process responds to variations of the initial mass distribution, in our simulations we used distributions of embryos derived by the following initial spatial and mass conditions for the dust component of the protoplanetary disk:

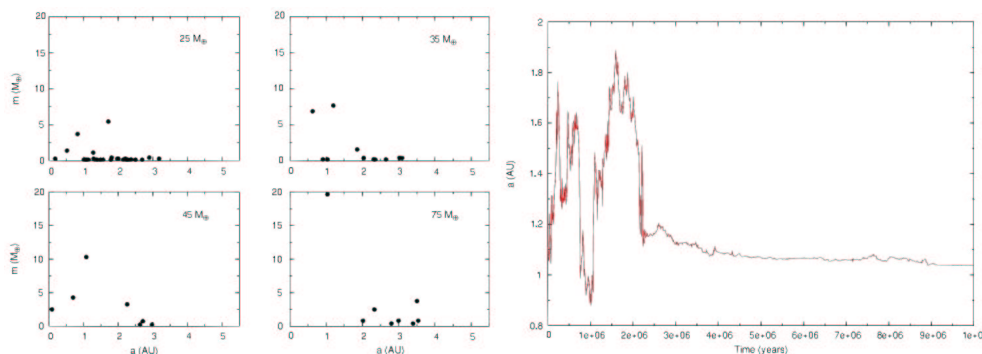
- superficial mass density  $\sigma \propto r^{-\frac{3}{2}}$ ;
- superficial mass density  $\sigma \propto r^{-1}$ ;
- superficial mass density  $\sigma$  constant;

- embryos spacing proportional to mutual Hills radii;

with  $r$  distance from the star. For all cases we considered four values of the total mass of the swarm, corresponding to two values of the dust surface density (50 and 100  $g\ cm^{-2}$ ) at the reference distance of 2.1 AU and to two values of the accretion efficiency (50% and 75%) in converting planetesimals into embryos. These superficial density values are derived from the estimates for the formation of the planetary companion in 47 UMa (Bodenheimer et al., 2000) and are one order of magnitude greater than those typically assumed for the minimum mass solar nebula. The latter values are so low that the time a embryo would need to accumulate the critical mass would be longer than the gaseous component lifetime (whose upper limit is about  $10^7$  years). At the beginning of the simulations, the embryos have eccentricities lower than 0.04 and inclinations lower than  $1^\circ$  with respect to the orbital plane of the binary system. In the simulations the embryos evolve dynamically under a symplectic mapping adapted by Chambers et al. (2002) to the S type binary star systems (referred to as *wide binaries* by the authors) for  $10^7$  years.

## 3. Results

The results of our simulations (summarized in Table 1 and in the left panel of Fig. 1) show a strong correlation between the fraction of cores reaching the critical mass before the dispersion of the disk gaseous component, the mass of the cores at the end of the simulations, the time necessary to reach the critical mass and the total mass initially incorporated in the swarms of embryos. With the only exception of the  $\sigma \propto r^{-\frac{3}{2}}$  case, we also found that the critical



**Fig. 1.** : Each graph in the left panel represent the typical situation at the end of a simulation with a specific value of the initial mass of the embryos swarm. The right panel shows the radial migration of a core during the accretion process.

core never forms at the right distance (about 2 AU). It usually forms at about 1 AU from its primary star and even in the case the core initially forms at the right distance, the gravitational scattering with the less massive bodies causes it to migrate radially to an internal orbit (see the right panel of Fig. 1).

#### 4. Dynamical stability in $\gamma$ Cephei system

We investigated the dynamical stability of the giant planet in  $\gamma$  Cephei by using the Frequency Map Analysis method by Laskar et al. (1992), originally developed to study the minor bodies of the Solar System. We tested the reliability of this method when applied to a binary star system by studying the elliptical complete three body problem, with continuous changes of the semi-major axis of the planet and its orbital eccentricity, over the entire lifespan of the primary star (set to, say,  $3 \cdot 10^9$  yr).

Our results, shown in Fig. 2, are in good agreement with the ones presented in the literature (Holman & Wiegert, 1999). They also agree with the numerical simulations we performed for the region close to the boundary of the stability zone, thus confirming the validity of the FMA method in such dynamical scenarios.

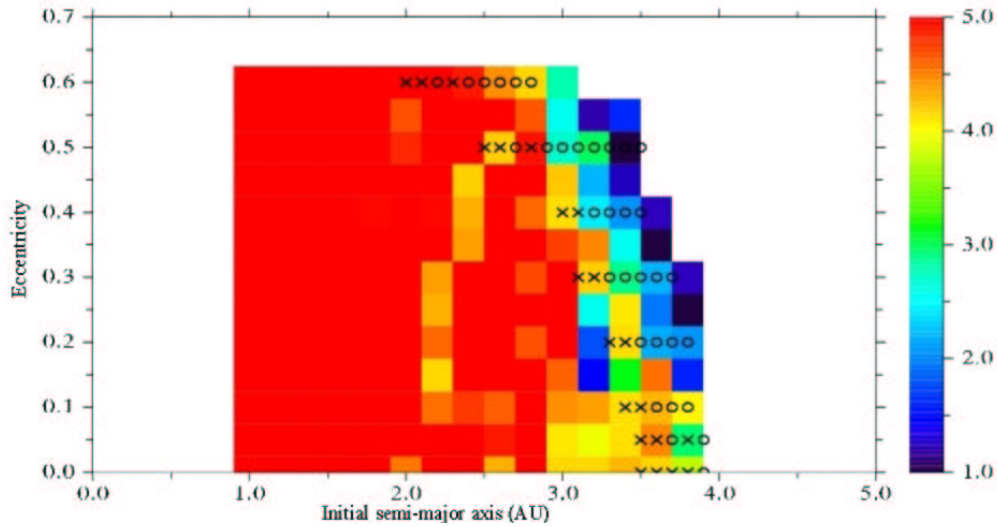
#### 5. The simulation and analysis software

For our simulations we used the **DPI** code. This is a set of Fortran77 subroutines which has been developed as a plug-in for the 6.2 release of J.E. Chambers' Mercury software (Chambers, 1999). **DPI** allows to use the symplectic mapping in a modified version for binary stars systems. Actually, only the wide binary algorithm (Chambers et al., 2002) has been implemented. We tested the reliability of our implementation by comparing the results of the test simulations done by Chambers et al. (2002) with the ones produced by our code. We also compare the output of our simulations for the stability study with that of the HJS algorithm by Beust (2003). **DPI** is freely obtainable by writing to the author at the e-mail address specified in the front page of this paper.

The algorithm we used for our stability analysis is part of the **ORSA** software, which can be found and downloaded at <http://orsa.sourceforge.net>.

#### 6. Conclusions

Our results seem to point out that the core-accretion model can be invoked to explain the formation of the giant planet in  $\gamma$  Cephei system if we assume a massive protoplanetary disk. While the timescale of the process does not seem to be a major problem, the final or-



**Fig. 2.** : In this figure we represent the dynamical stability of the giant planet in  $\gamma$  Cephei, with semi-major axis on the x axis and the eccentricity on the y axis, as resulting from the FMA. The color scale represent the diffusion velocity in the phase space and goes from blue for the chaotic regions to red for the stable ones. The X and O symbols represent the results (respectively stable and chaotic orbits) of the numerical integration close to the boundary of the stable region, performed with HJS and DPI.

bit of the planet shows a strong dependency from the initial mass distribution assumed for the solid component of the disk. Nevertheless, for the  $\sigma \propto r^{-\frac{3}{2}}$  dust density profile the planet resides near the observed orbit, which from our results turns out to be well inside the dynamically stable region of this stellar system.

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