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Evolutionary optimization of deflection missions with fly-by manoeuvre

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Abstract. The Earth, as the other celestial bodies in Solar System, is continuously exposed to impact hazard with bodies coming from space. The goal of this study is to find the optimal solution for a PHO (Potentially Hazardous Object) deflection mission with a kinetic impactor. An evolutionary algorithm, which combines the results obtained in parallel by differential evolution (DE), genetic algorithm (GA) and particle swarm optimization (PSO), is employed. Once identified a reference PHO, it has been considered an Earth-Asteroid mission that exploits the high mass of Jupiter to obtain a free velocity change performing a fly-by of the planet. The higher relative velocity at impact involves a higher deflection effect on PHO's trajectory. Spacecraft's direct and retrograde motion has been considered.

Key words. Potentially hazardous object – Deflection missions

1. Introduction

Taking a look at the surface of the Moon, it is immediately clear that craters caused by impacts with other bodies coming from space characterize this celestial body. Also, small bodies of different sizes continuously hit the Earth: 65 million years ago, an asteroid of 10 km of diameter hit the Yucatan peninsula and dug the Chixulub crater (170 km diameter), causing the extinction of dinosaurs and of many other species. Comets and asteroids are two typologies of objects potentially hazardous for the Earth. While the first ones are known from ancient times, for the discovery of the second ones it was necessary to wait the XIX century with the improvement of the spyglass, which was invented 200 years before. With a first look it is possible to distinguish these two types of celestial bodies: comets are objects apparently endowed with own light and/or with their characteristic tails, whereas asteroids appear as light points like stars. Their visible difference reflects partially their geological characterization. Asteroids typically are made of rocks and metals without atmosphere; comets, instead, are made of volatile components (as ice) that, if warmed up, produce a temporary atmosphere around the nucleus, generating characteristic tails. Near-Earth asteroids (NEAs) are catalogued as Apollo, Aten, Amor and IEO (Inner Earth Orbit) based on the characteristics of their orbit (see Table 1).

In recent times, the necessity of studying these bodies, of cataloguing them in function

Table 1. NEO definition

Class	Properties	Numerousness
Apollo	Semi-major axis > 1.0 AU Perihelion < 1.02 AU Earth crossing	62%
Aten	Semi-major axis < 1.0 AU Aphelion ≥ 1.0167 AU Earth crossing	6%
Amor	1.02 AU ≤ perihelion ≤ 1.3 AU	32 %
IEO	1.02 AU ≤ aphelion < 0.983 AU	6 knwon

of their hazardousness (21% of NEOs are considered to be potentially hazardous) and of understanding how it is possible to act in order to avoid the risk of impact with the Earth has been realized. As an example, a body larger than 140 m may cause damages on a regional scale, above 300 m the impact area would be to a sub-global scale; a global effect would occur with a body larger than 1 km. The impact with a PHO larger than 10 km may cause an extinction-class event. To define if a NEO (Near Earth Objects) could be considered a PHO (Potentially hazardous object) its MOID (Minimum Orbit Intersection Distance) and Absolute magnitude H must be evaluated. An object characterized by MOID < 0.05 AU and H < 22 is considered a PHO. Since 1999 scientists have adopted also the Torino Scale that ranks PHO's impact hazardousness evaluating some parameters as asteroid's diameter, impact energy, or deaths expected. If the impact probability is greater than 1%, the collision hazard must be taken into account and some defence strategies should be considered. In support of the Torino Scale, the Palermo Scale aids to define the object's observation and analysis priority.

Different mitigation strategies have been proposed that can be divided into two main groups depending on interaction techniques adopted. In both cases, velocity change causes an orbit variation in order to increase the miss distance between Asteroid and Earth at the close approach. Impulsive techniques (Kinetic Impactor, Explosions - conventional or nuclear) modify instantly the PHO's momentum, whereas Slow Push techniques (Focused Solar, Pulsed Laser, Mass Driver, Gravity Tractor, Asteroid Tug, Enhanced Yarkovsky Effect) apply a continuous thrust to the PHO for a long period.

The effect which is sought is to change the asteroid energy and orbital period, so as the asteroid arrives at the envisaged impact at a different time, thus missing the Earth. It's clear that the asteroid deflection must occur at a proper time before the impact and in the direction that maximizes the effect. In this study, the distance between asteroid and Earth at the moment of impact is the variable to be maximized. An optimization procedure based on hybrid evolutionary algorithms has been adopted. The method combines solutions obtained by three optimizers (based on a genetic algorithm, GA, differential evolution, DE and particle swarm optimization, PSO) which work in parallel according to the island model. Each algorithm acts on a defined population and the best solution obtained migrates to the other after prescribed intervals. In order to obtain a useful deflection using a kinetic impactor, a Jupiter flyby mission has been studied. The big mass of the planet can help the spacecraft to acquire a high velocity without fuel consumption and so increase its impact energy. An analysis of the influence of Jupiter's position has been carried out in order to determine the best one to perform the mission.

2. Evolutionary algorithms

Evolutionary algorithms are a family of stochastic methods based on the idea of natural evolution of species theorized by Darwin in 1859. The goal of EA is to select the best solutions by mimicking the natural processes, such as natural selection and the principle of survival of the fittest. Solutions evolve in a stochastic way, but the specific rules of the algorithm guide the evolution in order to obtain the fittest solution, that is, the optimal one. To evaluate the solutions' quality, an objective function called fitness function is introduced. The first generation comprises individuals generated randomly that represent candidate solutions of the problem. The fitness function of any of these elements is evaluated and the new solutions are created basing on fitness rank. The new generation is made up starting from the old individuals, which evolve through a limited number of stochastic operators.

2.1. Genetic algorithm (GA)

Genetic algorithms Holland (1975); Mitchell (1996) aim at replying the biological evolution through the mechanisms of reproduction, mutation and survival of the fittest to obtain the best solution. In the selection phase, through the evaluation of the fitness function, the best solutions are chosen to constitute a "parents' population". Then, reproduction operators split two or more individuals and combine them to create new individuals. Mutation allows modifying randomly any part of the solution.

For a given performance index either to be maximized or minimized, the algorithm's structure can be resumed as follow:

1. random creation of a starting population of solutions N_i;
2. evaluation of φ for each solution and assignment of a fitness value to each individual;
3. selection of parent solutions according to their fitness;
4. reproduction through solutions recombination. It's possible to apply a mutation operator in order to have in children's generation new features that in the parent's generation are missing.

Selection operators that can be used to select the parent's population solution are: Tournament selection: it consists to evaluate fitness function of two random chosen individuals. The winner is placed in parent population, the other one is removed. Roulette wheel selection: any individual's fitness function comprised in the population is evaluated and a probability to become parent is assigned proportionally to its fitness. In this way both good and bad solutions are considered in a different percentage, but the possibility to be used is non-zero. Through some techniques it is possible to improve the quality of the solution. It may occur that in the new generation some individuals are worse than the old ones. In this case it is possible to replace them with the best elements of the previous generation. This operation is based on Elitism principle. In order to have an initial population already with selected elements, it is possible to create a first group larger than the necessary and to choose the best individuals to compose the first generation. If solutions are focused in a restricted area of the domain and fitness function does not improve, it is possible to cause a mass extinction and replace individuals with new ones. To assure the variety of genetic heritage, in addition to mutation is possible to use cross-over operator that replace one or more segment of the solution with sections taken from another individual. In the real-code formulation adopted here, the new generation is created exploiting a probability distribution defined as:

$$P(\beta) = 0.5(\eta + 1)\beta^{\eta} \qquad \text{for } \beta \leq 1 \qquad (1)$$

or

$$P(\beta) = 0.5(\eta + 1)/\beta^{(\eta+2)} \qquad \text{otherwise} \qquad (2)$$

Parent values x_1 and x_2 are used to determine new individuals y_1 and y_2; a random value

$0 < u < 1$ is first chosen and then $\bar{\beta}$ is defined so as:

$$\int_0^{\bar{\beta}} P(\beta)\mathrm{d}\beta = u \quad (3)$$

Children are computed as follow

$$y_1 = 0.5[(x_1 + x_2) - \bar{\beta}|x_2 - x_1|] \quad (4)$$

$$y_2 = 0.5[(x_1 + x_2) + \bar{\beta}|x_2 - x_1|] \quad (5)$$

2.2. Differential evolution (DE)

Differential Evolution Storn & Price (1995) through feature of already existent individuals, creates a new population. The value of an optimization variable for a new element is defined as:

$$y = x_1 + F(x_2 - x_3) \quad (6)$$

where x_1, x_2 and x_3 are the corresponding values of random elements of the current generation and F is a scaling factor. DE takes advantage of the sparseness of the initial population to converge to the optimum. At each step, solutions approach the optimal one and the research domain become smaller. The algorithm is featured by the following steps:

1. Setting up of a initial population consisting of N_i individuals (real limited values);
2. New y individuals generation;
3. Comparison between a new element (y) and an old one (x). Both best and worst solutions are chosen (the first in a higher percentage). In this way, the new population comprises a larger genetic heritage of the best individuals;
4. Return to step 2 until the imposed limit is reached.

Another formulation (not used in the present work) of new individual that can be adopted is the following:

$$y = x_1 + F(x_2 - x_3) + C(x_4 - x_5) \quad (7)$$

C, as F, is a scaling factor.

2.3. Particle swarm optimization (PSO)

PSO Eberhart & Kennedy (1995) is inspired to the behaviour of birds, fishes and insects swarms. This technique does not have a population that evolves, so it has no operators like cross-over or mutation, but is characterized by individuals that migrate in the problem domain, following optimum (local) particles. The element is defined by its position in space and this substitutes the genetic heritage information evaluated in previous techniques (GA, DE). The rules that characterize the algorithm are:

1. Separation: avoid collision between near particles;
2. Alignment: to target swarm's medium position;
3. Cohesion: to target neighbour's medium position.

Each particle is characterized by a velocity v. If a unit time interval is considered, the new individual can be defined as:

$$y = x + v \quad (8)$$

The idea is to change, for each step, the velocity, targeting the best solution found:

$$v_{new} = v_{old} + c_1 k_1[x_(p_{best}) - x] + \\ +c_2 k_2[x_(g_{best}) - x] \quad (9)$$

where $x(p_{best})$ is the particle's best position reached, $x(g_{best})$ is the global best position reached, c_1 and c_2 are learning factors that depend on adopted strategy, k_1 and k_2 are random numbers in the [0,1] interval. PSO is significantly different with respect to the other two techniques here presented (GA, DE). In DE and GE, solutions move to the global optimum thanks to the share of information between all elements. In PSO, only the best solution shares information with the others. Compared with the others, PSO reaches the optimum quickly, in most cases.

3. Statement of the problem

It has been supposed that, at time t_3, a PHA, characterized by a_a = 1.1 AU and e_a = 0.1

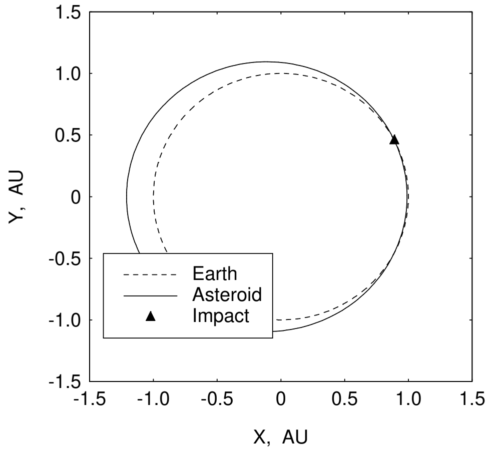

Fig. 1. Orbit geometry and impact point.

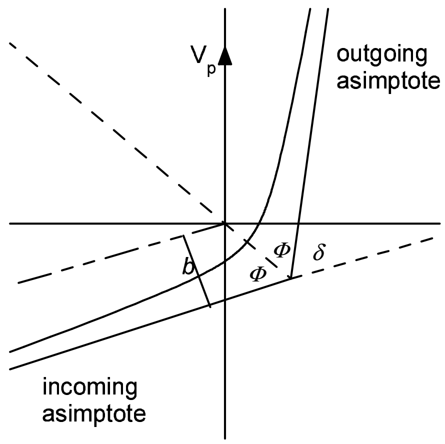

Fig. 2. Flyby geometry.

(Apollo type) may collide with Earth at the orbit intersection point in the first quadrant (see Fig. 1). The asteroid has a 200 m diameter and its weight is assumed to be 10^{10} kg. Earth's and Asteroid's orbits are co-planar. Earth is assumed to move on a circular orbit (r = 1 AU). Goal of this study is to give to the selected PHA a velocity change ΔV, through an impact with a high mass spacecraft (kinetic impactor technique) that causes a variation of the asteroid's orbit in order to avoid collision with the Earth.

3.1. Mission description

The patched-conic approximation is adopted. Time spent inside planets' sphere of influences and their dimensions are small in comparison to the mission scale and can therefore be neglected. Only the heliocentric trajectory is considered with proper boundary conditions at planetary encounters. It's supposed that the spacecraft initially moves on a parking orbit at 200 km altitude, submitted only to Earth's gravity. The spacecraft's velocity is

$$v_{sc} = \sqrt{\frac{\mu_E}{r_E + h}} = 7.778\text{km/s} \tag{10}$$

where μ_E is Earth's gravitational parameter, R_E identifies Earth's radius and h is parking orbit altitude. At a proper time, rockets are switched on and the spacecraft moves on a hyperbolic trajectory to the limit of the Earth's sphere of influence, reaching v_∞ (hyperbolic excess velocity) aligned with the asymptotes of the hyperbola. This is the instant, called t_0, wherein spacecraft's motion becomes heliocentric, and the spacecraft moves towards Jupiter, which is reached at time t_1. Gravity assists is used to make the spacecraft acquire, without fuel consumption, high inclination of to make its orbit retrograde, as shown in the following.

Flyby geometry is described in Fig. 2 and 3. The spacecraft's velocity when entering Jupiter's sphere of influence (subscript -) is $V_{\infty-} = V_- - V_P$ where V_- is spacecraft's heliocentric velocity before the fly-by manoeuvre and V_P is Jupiter's velocity.

The spacecraft's velocity after the manoeuvre (subscript +) is $V_+ = V_{\infty+} + V_P$ and $\delta = \pi - 2\phi$ is the fly-by rotation angle defined as

$$\cos\phi = \sin(\delta/2) = 1/e \tag{11}$$

Maximum allowable rotation has been taken into account by limiting the height of the flyby above Jupiter's surface above 250000 km; ΔV_{FB} is introduced when this constraint is violated and it is necessary to switch on the rockets to perform the manoeuvre. Space probe gets into a new elliptic trajectory and impacts

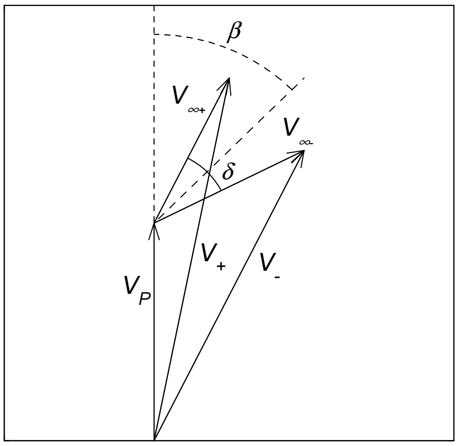

Fig. 3. Flyby velocity vectors.

the asteroid at time t_2. Then, the asteroid flies a modified trajectory until time t_3 when the miss distance is evaluated.

3.2. Model description

The total velocity change is the sum of ΔVs performed at each step of the mission. Spacecraft's mass on the starting LEO is assumed to be $m_{LEO} = 10^4$ kg. So, by using Tsiolkowsky's equation

$$m_0 = m_{LEO} \exp(-\Delta V_0/c) \tag{12}$$

In a similar way the mass which may be spent during Jupiter's flyby is evaluated and the spacecraft mass at asteroid deflection is

$$m_f = m_0 \exp(-\Delta V_{FB}/c) \tag{13}$$

To compute the minimum Earth-asteroid distance which is achieved after deflection, the variation of asteroid's velocity has been linked to alteration of asteroid's orbital elements caused by the perturbation. In this way it's possible to define the change of asteroid's radius with respect to the unperturbed status. A little deviation is expected, so to approximate asteroid's position at t_3 (instant of presumed collision with Earth), that is a function of the velocity change caused by spacecraft impact at t_2, a linearized method has been used. Assumed that

$$\Delta V = \frac{m_{SC}}{m_{AST}}(V_{SC} - V_{AST}) = \frac{m_{SC}}{m_{AST}} V_{rel} \tag{14}$$

and defining velocity change in tangential, normal and out-of-plane coordinates $\Delta V_2 = [\Delta v_t, \Delta v_n, \Delta v_h]^T$ regrouping orbital element in the vector $\alpha(t) = [a, e, i, \Omega, \omega, M(t)]^T$ it is possible to write $\Delta\alpha(t_2) = G(t_2)\Delta v_2$

To evaluate asteroid's position at t_3, it's necessary to compute the change of mean anomaly caused by the impact with the spacecraft: $M(t_3) = M(t_2) + n(t_3 - t_2)$ This can be seen as the sum of changes due to mean anomaly variation at the impulse and to the secular change caused by angular velocity alteration: $\Delta M(t_3) = \Delta M(t_2) + \Delta n(t_3 - t_2)$ where $\Delta n = -3n/2a\Delta a$, so $\Delta\alpha(t_3) = G(t_3)\delta v_2$ Note that mean motion change at impact depends on asteroid's angular velocity variation on the perturbed trajectory and on its semi-major axis change. Position change in radial, tangential and out-of plane coordinates can be written as $\Delta r_3 = [A]\Delta\alpha(t_3) = [A][G(t_3)]\Delta v_2$ [A] represents relative motion equation, obtained evaluating partial derivatives of the position vector with respect to orbital elements. The heliocentric position change Δr_3 is used to evaluate the asteroid position change ΔR_3 in a reference system centred at the Earth and its components in radial ΔR_R, tangential ΔR_T and out-of-plane ΔR_N directions. The distance between asteroid and the Earth can be defined as

$$D_1 = \sqrt{(R_3 + \Delta R_R)^2 + \Delta R_T^2 + \Delta R_N^2} \tag{15}$$

It is important to remark that D_1 is not the minimum distance, as the asteroid may reach the minimum distance at a different time and can therefore get closer to Earth. Exploiting a linear motion approximation, one can notice that ΔR_T (relative motion change along relative velocity direction) doesn't contribute to miss distance change, as shown in Fig. 4, and therefore

$$D_2 = \sqrt{(R_3 + \Delta R_R)^2 + +\Delta R_N^2} \tag{16}$$

In this study R_3 is assumed null.

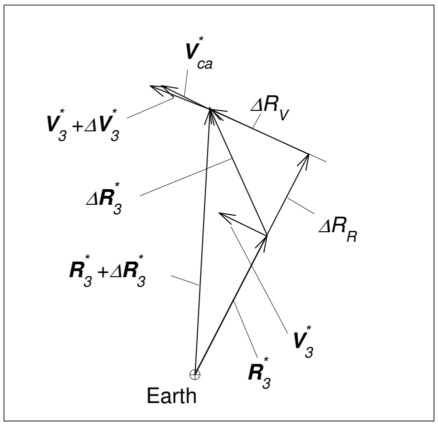

Fig. 4. Close encounter geometry and miss distance evaluation.

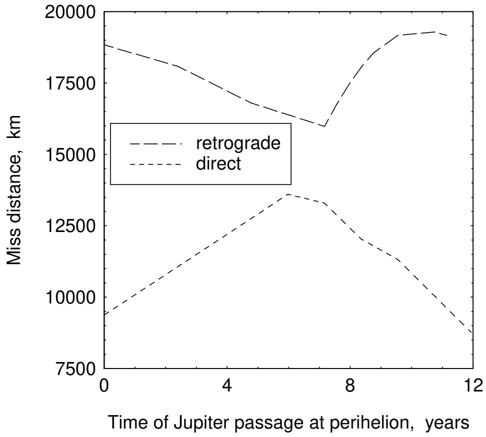

Fig. 5. Influence of Jupiter position on maximum achievable miss distance.

The optimization procedure is made up as follows. For each algorithm, the number of population element is initially set, in particular: 100 elements for GA 30 elements for DE 30 elements for PSO and domains for the optimization variables are defined. Variables to be optimized are three: t_0, Δt_1 and δt_2 where t_0 is the starting instant, δt_1 is the duration of the first leg (the one that describes the trajectory from Earth to Jupiter) and δt_2 is the interval of the second leg (from Jupiter to asteroid). Starting from values of the optimization variables, the spacecraft's trajectory parameters are evaluated by solving Lambert's problem given time and position at the arc extremities, and velocities at the beginning and at the end of each trajectory leg are evaluated. Then, velocity changes, spacecraft mass at deflection, asteroid velocity change and miss distance are evaluated. The solution that guarantees the maximum D_2 is therefore found. The method is iterative, so a new population is generated until the user considers appropriate to end the procedure (solution convergence is noticed).

4. Results

In this study, the deflection, with a kinetic impactor, of a PHA that may impact Earth has been computed, by means of a hybrid evolutionary algorithm. The spacecraft, with a high mass collides at time t_2 with the asteroid with high relative velocity acquired also thanks to a Jupiter fly-by manoeuvre, to avoid impact with Earth. Considering that Jupiter's revolution time is around 11.86 years, an analysis of the influence of Jupiter position on its orbit has been carried out, in order to find the best moment to perform fly-by manoeuvre (in terms of velocity increase). Jupiter perihelion time of passage has been altered to study this effect. Besides mission with direct motion, also retrograde motion after Jupiter's flyby has been considered. Starting mission time in the range between 9.55 and 19.1 years before the envisaged impact has been considered.

Analyzing results, it's noticeable that Jupiter passing time at perihelion does not influence significantly performance. Worst results give minimum distance in a range between 8766.8 km and 15981.6 km that are absolutely acceptable. Best solutions for both cases are presented in Table 2, where t_{ppJ} is the Jupiter passing time at perihelion, t_0 is the mission starting time, t_1 is the fly-by time, t_2 is the impact time, δ is the fly-by angle deviation, V is the impact relative velocity and D_2 is the distance between Earth and Asteroid, computed at Earth-Asteroid presumed impact time, after

Table 2. Results for the best performing missions with Jupiter's gravity assist

Solution	t_{ppJ}	t_0	t_1	t_2	δ,	V_{rel}, km/s	D_2, km
direct	5.97	18.35	1.23	4.61	59.78	49.88	13602.8
retrograde	10.74	18.79	1.09	4.79	78.20	68.61	19286.5

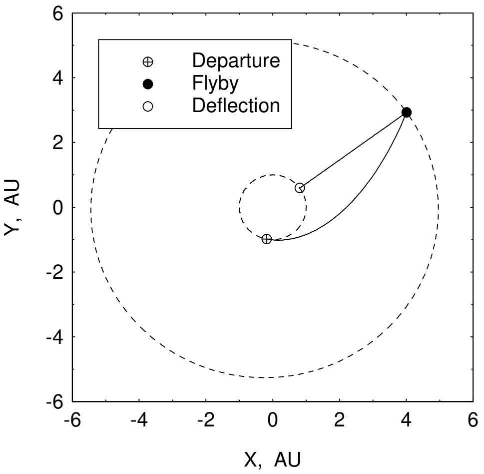

Fig. 6. Best solution for direct orbits.

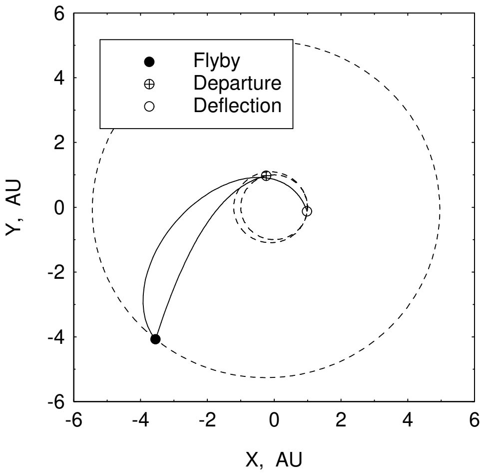

Fig. 7. Best solution for retrograde orbits.

the deviation. All time values are expressed in years. For the sake of comparison, results from previous studies focused on direct mission are reported in Table 3: the spacecraft leaves the parking orbit and impacts the asteroid without any impulse and any other manoeuvre.

It's noticeable that best results are obtained with Jupiter fly-by manoeuvre, both in terms of impact velocity and minimum distance. Comparing direct mission and direct motion fly-by mission, velocity increase is around 132% and compared with retrograde motion fly-by mission velocity increases around 229%.

The best solutions are plotted in Fig. 6 and Fig. 7. If only direct trajectories are considered, Jupiter flyby inserts the spacecraft into a polar orbit to increase the relative velocity at deflection. Additional velocity increase is obtained with a retrograde trajectory. In both cases, no consumption of fuel is needed to perform fly-by manoeuvre thanks to the high value of the gravitational constant of Jupiter.

5. Conclusions

This study aims to evaluate the efficiency of the deflection of an asteroid in course of collision with the earth, with the kinetic impactor technique; the deflection mission is optimized by using a hybrid evolutionary algorithm (based on differential evolution, genetic algorithms and particle swarm optimization that work in a cooperative way). To perform trajectories analysis a method based on Laplace model and patched conic approximation has been adopted. The evolutionary algorithm adopted demonstrates its high speed convergence to the best solution and its robustness. Only few minutes are required to perform the analysis on a standard PC. The solutions show the high efficiency of the deflection strategy, which moves the asteroid away from Earth for a distance

Table 3. Results for the best performing mission without gravity assist

Solution	t_0	t_2	δ,	V_{rel}, km/s	D_2, km
direct	18.08	17.41	156.6	6.08	5757.8

comprised between 13602 km and 19286 km (2.31 and 3.03 times Earth's radius). The optimization procedure is highly flexible and more complex missions with multiple flybys (Venus, Earth) could be treated with the same method.

References

Bally, J., Morse, J., & Reipurth, B. 1996, in Science With The Hubble Space Telescope – II, Benvenuti P., Macchetto F.D., Schreier E. J. eds. (Space Telescope Science Institute, Baltimore), 491

Bate, R.R., Mueller, D.D., & White J.E. 1971, Fundamentals of Astrodynamics, (Dover Publications, New York)

Curtis, H. 2009, Orbital Mechanics for Engineering Students, (Butterworth-Heinemann, Burlington)

Eberhart, R., & Kennedy, J. 1995, in Sixth International Synposium on Micro Machine and Human Science (Institute of Electrical and Electronics Engineers, Piscataway, NJ), 39

Di Martino, M. et al. 2009, ESA-SP 1310

Farinella, P., Paolicchi, P., & Zappala, V. 1983, Gli Asteroidi (Il Castello, Milano)

Goldberg, D., & Deb, K. 1991, A Comparative Analysis of Selection Schemes Used in Genetic Algorithms, in Foundations of Genetic Algorithms, ed. by Rawlins G.J.E. (Morgan Kaufmann, San Mateo, CA), 69

Holland, J. 1975, Adaption in Natural and Artificial Systems (University of Michigan Press, Ann Arbor, MI)

Izzo, D. 2007, Journal of Guidance, Control and Dynamics, 30, 401

Larson, W.J., & Wertz, J.R. 1999, Space Mission Analysis and Design, 3rd edition, (Kluwer, Boston)

Mitchell, M. 1996, Introduction to Genetic Algorithms (Massachusetts Institute of Technology Press, Cambridge, MA)

NASA 2007, Near-Earth Object Survey and Deflection Analysis of Alternatives, Report to Congress, March 2007

Park, S.Y., & Ross, I.M. 1999, Journal of Guidance, Control and Dynamics, 22, 415

Rigutti, M. 1997, Comete, Meteoriti e Stelle Cadenti (Giunti, Firenze)

Rosa Sentinella, M.R., & Casalino, L. 2009, Celest. Mech. Dyn. Astr., 105, 211

Rosa Sentinella, M.R., & Casalino, L. 2008, Journal of Spacecraft and Rockets, 46, 365

Schaub, H., & Junkins J.L. 2003, Analytical Mechanics of Space Systems, (AIAA, Reston, VA)

Storn, R., & Price, K. 1995, Differential Evolution - A Simple and Efficient Adaptive Scheme for Global Optization over Continuos Spaces (International Computer Science Institute, Berkeley, CA), ICSI TR–95–012