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Boscovich as an engineer: the statics of masonry domes

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Abstract. The collection of writings by Ruggiero Boscovich contains a certain number of studies that can be considered of "engineering" character, mostly focusing on problems of hydraulics and structural mechanics. Nevertheless, such studies hardly can be regarded as part of Boscovich's direct interest. They were usually meant at answering specific questions, when Boscovich was acting as a consultant for people that were facing serious problems of different kind and asked his advise, considered as precious because of the prestige that Boscovich enjoyed in his time.
In this paper attention is focused on one problem, the statics of masonry domes, which Boscovich faced twice in two different contexts. In these studies he employed, to my knowledge for the first time for computation purposes, a failure mechanism that at the end of the century became the basis for systematic and rigorous methods for the analysis of arches, vaults and domes. Boscovich work can be regarded as anticipating these results.

Key words. Masonry Domes – Failure mechanisms – St Peter cathedral in Rome – Milan cathedral

1. Introduction

The National (Italian) Academy of Sciences has undertaken the program of collecting all Boscovich written work. This *Edizione Nazionale Boscovich* includes, beside a huge amount of correspondence, about 150 printed documents and more or less 25 of them refer to problems of structural mechanics and hydraulics (Proverbio 2007). Even neglecting contributions on subjects such as surveying, it appears that a non negligible percentage of Boscovich work refers to applications of engineering nature.

However, these studies look in a sense anomalous within Boscovich's activity. In contrast to papers on astronomy, mathematics, geometry, natural philosophy and of literary nature, which are obvious manifestations of his main interests, they were developed to answer questions that were brought to him from outside. The deepness and the vastness of his knowledge were held in such esteem among his contemporaries that his opinion was requested on several problems of importance, such as harbors to be reactivated, channels and roads to be renewed or constructed, structures to be repaired. In all instances Boscovich provided fully satisfactory answers, showing an

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Fig. 1. Report containing the minutes of the controversy on the management of the Bientina lake water. The report starts with the description of the problem and ends with the concluding remarks by father Ximenes. In between there are considerations and rebuttals of a few scholars, including Boscovich.

outstanding attitude in dealing with practical problems.

This strengthened his reputation and prestige and reiterated the number of requests for his advice, so that Boscovich's services were in demand in several places. Significant under this respect is the mission he was entrusted by the Republic of Lucca, who sent him to the court of Vienna as his representative in a controversy against Tuscany about the draining of the lake of Bientina (Fig. 1). The intervention envisaged by father Leonardo Ximenes, mathematician of the duke of Tuscany, bred some fear, in that the outflow water of the lake was thought to threaten the territory of the Republic and the city of Lucca itself. In his mission Boscovich was not supposed to limit himself to technical aspects, but was in charge of managing the controversy in behalf of the Republic of Lucca, a role in which diplomatic capabilities were as important as technical knowledge. He managed this task with such skill that the Luccans made him a honorary citizen.

As a whole, this "consulting" work is significant since it underlines an important aspect of Boscovich's personality and emphasizes his versatility and the broad spectrum of his knowledge. The results reached, however, are conditioned by the rather specific problems dealt with and these studies do not possess the generality and breadth that is typical of Boscovich's main production. Nevertheless, the assumptions made, the rigor employed and the methodologies introduced often are of great interest and many of these studies still must be considered as outstanding scientific contributions.

To this category definitely belong the three documents referring to masonry domes. Boscovich faced the problem in two different situations. The first was when a number of extensive cracks appeared in the dome of St Peter cathedral in Rome, causing well understandable concern in the entourage of Pope Benedict XIV. Three outstanding mathematicians of the time were asked to study the problem and to propose remedies, and Boscovich was one of them. The second was connected with the construction of the main pinnacle of Milan's cathedral, which was to be placed on the top of the dome. This was a different problem, since the dome itself was in good health. The concern was about its capability at withstanding a significant additional load.

In both instances the static behavior of the dome had to examined and the margin with respect to failure had to be assessed, a problem that at the time was tackled with rather rough methods, which were rapidly found to be inadequate. Boscovich envisaged new strategies that, even if mostly based on ingenuity, are interesting in that they anticipate a method developed by Coulomb and Mascheroni at the end of the century, which became the fundamental tool for the study of masonry constructions. In this paper these aspects are briefly discussed.

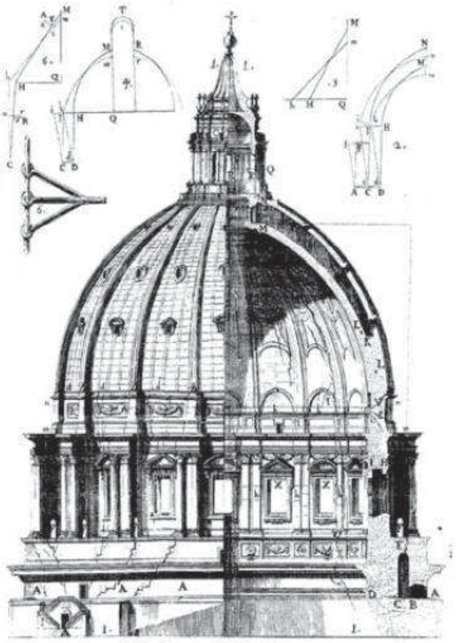


Fig. 2. The situation of the dome of St Peter at the beginning of 1742, with widespread cracks clearly visible (Le Seur, Jacquier & Boscovich 1742).

2. The dome of St Peter and the three matematicians

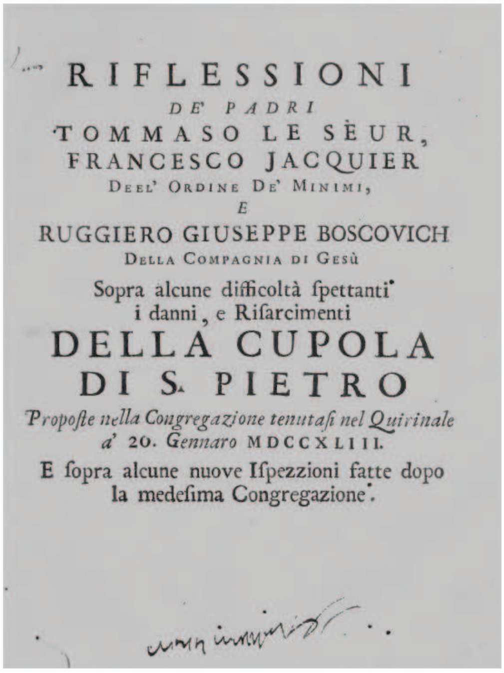
Indications of structural trouble in the dome of St Peter in Rome appeared immediately after its completion. Initially minor, unpleasant symptoms became increasingly worrying with time and in 1740 the number and the extension of cracks and signs of damage detected were large enough to create a justified concern. Pope Benedict XIV, who looked with favor to the scientific developments of the time, ordered a number of inspection and finally asked three highly considered mathematicians, Thomas Le Seur, Francois Jacquier and Boscovich, to judge the status of the monument and to suggest possible remedies.

The situation the three mathematicians had to face is illustrated in Fig. 2. The dome was damaged almost everywhere. In the picture, the most visible cracks are at the base of the drum, but no lesser concern was induced by those running along the meridians. The document produced (Le Seur, Jacquier & Boscovich 1742) depicted in a rather pessimistic fashion the current status of the dome and stated that some kind of intervention was demanded. More specifically the mathematicians suggested that large iron bands be put about the top of the drum to strengthen the original reinforcement that was probably deteriorated by corrosion and in any case insufficient.

The document was the object of several discussions and much criticism, partly because the pessimistic view of the mathematicians was not shared by everybody and partly for jealousy among peers. An enormous number of written opinions were produced and the debate was about to degenerate into a quarrel. A convention was held at the beginning of 1743 and after it the three mathematicians produced a second report (Le Seur, Jacquier & Boscovich 1743), in which they essentially confirmed their previous conclusions (Fig. 3). The controversy ended with a solomonic decision: it was agreed that following the mathematicians' suggestions maybe was not necessary at the moment, but possibly helpful for the future and the iron bands were put in place.

The history of this debate is interesting, but lays outside the scope of this paper (details are provided by Capecchi (2010a), where the complete list of the documents produced can also be found). More relevant to present purposes is the technical content of the study and the way it was conducted, which are now discussed briefly, referring to the first report (Le Seur, Jacquier & Boscovich 1742).

The document starts with a careful description of the cracks detected and the assessment of the severity of their possible consequences. Probable causes are also examined and on this basis a number of failure modalities are envisaged and discussed in order to single out the one that most likely is responsible for deterioration. To this purpose, detailed computations are performed, based on data (geometry and weights) that are assessed to the best accuracy available at the time (though insufficient for present standards). Purpose of the computations was to establish whether resisting forces



RIFLESSIONI
DE' PADRI
TOMMASO LE SÈUR,
FRANCESCO JACQUIER
DELL' ORDINE DE' MINIMI,
E
RUGGIERO GIUSEPPE BOSCOVICH
DELLA COMPAGNIA DI GESÙ
Sopra alcune difficoltà ſpettanti
i danni, e Riſarcimenti
DELLA CUPOLA
DI S. PIETRO
Propoſte nella Congregazione tenutaſi nel Quirinale
a' 20. Gennaro MDCCXLIII.
E ſopra alcune nuove Iſpezzioni fatte dopo
la medeſima Congregazione.

Fig. 3. Cover page of of the second report (Le Seur, Jacquier & Boscovich 1743).

balanced the reversing loads. It was found that this hardly was the case, which justified the extensively damaged status of the dome and indicated that little margin was left in terms of additional resources. The proposal of circling the upper part of the drum with iron bands was advanced to prevent progressive deformation and increasing damage.

Obviously, conclusions were conditioned by a number of uncertainties on data and by approximations in the model, which were honestly pointed out. Nevertheless, the problem was handled with admirable logic and rigorous methodology, each step was deeply criticized to assess its merit and detect possible weak points, assumptions were thoroughly discussed and a number of interestingly new points were raised. As a whole, the document produced by the three mathematicians possesses all the features characterizing an outstanding scientific contribution, at least for the time.

3. The main pinnacle of the Duomo di Milano

The *Duomo di Milano* (Milan's cathedral) is a rather peculiar monument. Conceived in a northern Europe gothic style, its construction started in 1386, when gothic architecture was at its end (at least in Italy), and lasted a few centuries. It follows that several significant parts of the monument were built in the renaissance, baroque and neoclassic periods and, even if great attention was always paid to preserve the homogeneity of the style, evidence of the new tendencies show up in several instances.

The dome (Fig. 4) was built at the end of the fifteenth century and conflicts with the gothic support beneath arose also at the statical level. The construction was the source of much apprehension among the builders and the administrators of the *Veneranda Fabbrica del Duomo*, the board responsible for the construction and the maintenance of the monument, and its completion was a significant achievement.

However, the *Duomo* was supposed to be completed with a pinnacle boldly pointing toward the sky and resting on the top of the dome. The problem of erecting such a pinnacle, a daring structure in itself, on such a troublesome support was really challenging and the *Fabbrica* considered it with understandable hesitation. An early attempt started shortly after the completion of the dome was immediately interrupted.

The problem was reconsidered only in the first half of the eighteenth century, with full awareness of the difficulties of the task. For about thirty years several studies were carried out and several proposals were advanced, until in 1764 Francesco Croce, the Architect of the *Veneranda Fabbrica*, produced a design that was considered as feasible and eventually accepted. In the relation accompanying his proposal Croce discussed in detail the statical aspects and how they were considered. This did not prevent that a considerable amount of perplexity spread among the public. Several written opinions were expressed, some in favor of and most against the proposal. A particularly violent attack was launched through

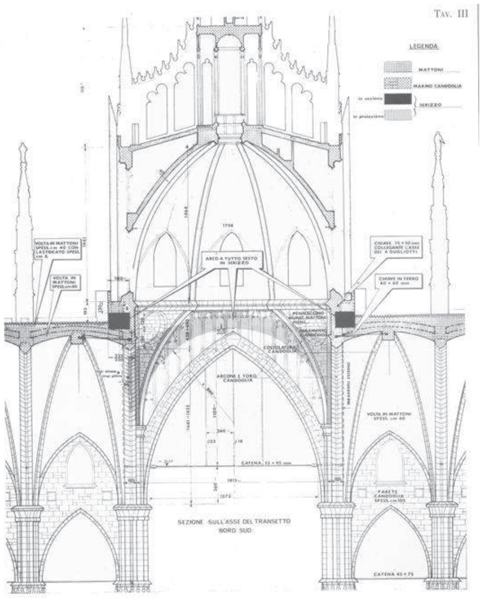


Fig. 4. Cross section of Milan's cathedral, showing the dome, the dome cladding and the lantern. The main pinnacle was built on the top of it

a pamphlet that, though anonymous, was immediately recognized to come from father Paolo Frisi, an outstanding mathematician and an eminent personality in Milan at the time (Masotti 1962). Actually, the attack seemed to be motivated more by a hostility for eighteen century architecture than by worries about the stability of the pinnacle and of the cathedral. The statical objections raised were rather specious and Croce had not difficulty in retorting each of them. However, the prestige that father Frisi enjoyed and the rather high level of widespread perplexity among the public demanded that some action be taken.

Since the early stages of the process the *Fabbrica* did recognize the opportunity of asking the opinion of external experts before taking a final decision. At this point Croce himself urged that this step be taken. As experts were chosen two mathematicians, Francesco De Regi and Boscovich, then in Milan, and count Benedetto Alfieri, Architect of the king of Sardinia, who declined the invitation proposing in his place the colleague Francesco Martinez. Subsequently, also father Beccaria was contacted to give his opinion about possible damage due to lightening. All the experts approved Croce's proposal and the pinnacle was completed by 1770 (Fig. 5).

A more comprehensive coverage of the events that caused Boscovich involvement can be found, e.g., in Stolfi (2003) and Capecchi (2010b). Attention now is devoted to the technical content of the experts work. Except that for father Beccaria, who was contacted for a very specific problem, the experts were asked a twofold question: whether *the pinnacle was sound enough in itself* and whether *it was equally sound with respect to the stability of the entire cathedral*. In fact the pinnacle, nearly thirty meters high and fully exposed to wind and storms, was in itself a daring piece of work, but also its support, the dome, constructed 250 years earlier overcoming considerable difficulties and with much anxiety, was a likely source of trouble. Actually, Croce's design was found to be fully adequate in itself, so that the first part of the question was dismissed quickly and all experts concentrated on the second point, regarded as the crucial one.

The three experts worked individually, producing three different documents. That of Martinez is considered of limited interest, while the contributions of the two mathematicians are of great significance. Boscovich and De Regi made their own measurements, introduced independent assumptions and used different methods and models for computations. As a consequence, they obtained different results, but both concluded that Croce's proposal would not jeopardize the stability of the cathedral.

As for the case of St Peter's dome, a number of uncertainties on geometry and material distribution were unavoidably present and drastic assumption had to be introduced to permit computations, so that results should be considered with caution. We know today that the conclusions of both mathematicians were too optimistic and that the main pinnacle, with its weight, is not far from exhausting the *Duomo* bearing capacity. Nevertheless, the two documents are admirable under many respects, especially that of Boscovich. His report (Boscovich 1765) possesses the same clarity

Fig. 5. The main pinnacle of Milan's cathedral. The dome is not visible from outside, being covered by a rather elaborated cladding.

and rigor as the previous studies on St Peter and also must be considered a scientific contribution of significant value.

4. Dome failure according to Boscovich

When Boscovich was carrying on his studies on domes, the assessment of the load bearing capacity of masonry structures was under study by several scholars, mainly in France. Available procedures required the definition of a suitable *failure mechanism*, i.e. a failure modality making evident the separation between reversing and stabilizing loads and permitting the establishment of a balance among them. At the time, a widely accepted method was that introduced by Belidor (Belidor 1734), an improved version of a previous proposal by De la Hire. The method was meant as a dimensioning procedure and when used to verify the behavior of existing structures gave rise to shortcomings, sometimes severe, that had been remarked in a few instances. Nevertheless it remained fairly popular and was almost universally used.

An outline of the method is now given, referring for simplicity but without lack of generality to a plane arch. Belidor conceived a failure mechanism with the central portion of the arch dropping downwards and pushing aside the remaining parts and the pillars (Fig. 6a). A number of arbitrary assumptions are made: separation occurs at a 45° degrees inclination with respect to the horizontal line and the upper part transmits to the lower part a force acting at the centroid of the separation section. Also, the force is perpendicular to the section, since friction is neglected. The effects of this *reversing* load should not exceed those of the *resisting* forces, the weight of the pillars and of the lower parts of the arch (Fig. 6b). Essentially, failure is supposed to occur according to a shear type mechanism, in which the different parts of the arch keep rigid individually and slide with respect to each other.

In his analysis of Milan's dome, father De Regi employed this method, but became immediately aware of its limitations. If applied in its original version, the method predicted not only that the *Duomo* could not bear the main pinnacle, but that it should not stand even as it was, in patent contradiction with everyday experience. De Regi had to abandon the original version of the method, but did not give up the method itself. By employing a great amount of ingenuity, he introduced some modifications that overcame the problem, establishing that the pinnacle could be built safely. The details of the actions taken are of no importance for this paper. More relevant is the fact that De Regi, even when faced with an absurd result, preferred to elaborate some aspects of the method rather than looking for alternatives, which stresses the popularity of Belidor's method and its widespread acceptance.

Boscovich attitude was different. The three mathematicians in Rome and Boscovich by himself in Milan were not conditioned by existing procedures. They faced the problem as it was and developed on this basis the suitable analysis strategies and computational models. Fig. 7 (an enlargement of a detail of Fig. 2) depicts the mechanism considered for the analy-

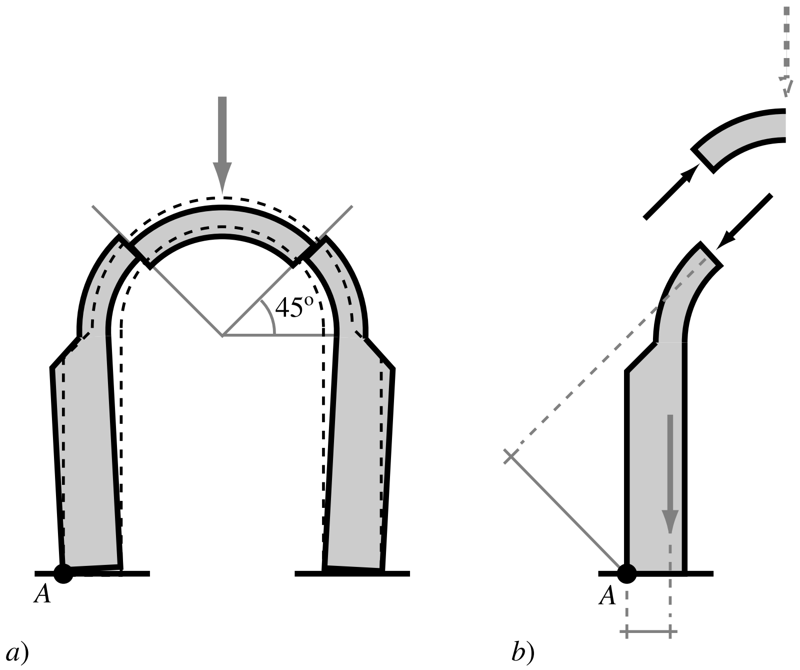


Fig. 6. The failure mechanism on which Belidor's method is based. Stability requires that the moment about point *A* of the reversing force (black) be more than compensated by that of the resisting force (gray).

sis of St Peter dome, selected after examining and discarding a number of alternatives. The mechanism still consists of rigid blocks, which however rotate, rather than slide, with respect to each other, a significant departure from a routine application of Belidor's method or *ad hoc* modifications of it.

Fig. 8 schematically shows the reversing and stabilizing loads, still indicated in black and gray, respectively. The horizontal force represents the in plane contribution of the circumferential reinforcement, which plays a key role in ensuring the dome stability and, in fact, according to the three mathematicians was to be increased. Its presence introduces a significant difference with respect to Belidor's procedure. While the latter operates in the plane and effectively identifies the dome behavior with that of an arch, the three-dimensional nature of the problem is now considered, in a simplified but essentially correct fashion.

In his study on Milan's cathedral Boscovich followed a similar path of reasoning, but the mechanism he arrived at was different, because of the different structural context. In St Peter the weak point was identified with the drum which, being severely cracked, was thought to behave as a collection of independent columns, liable to outward leaning. In the *Duomo di Milano*, on the contrary, possible outward motions of the bottom of the dome were contrasted by the naves, the transept and the apse body and this contrast was considered to be effective enough. Therefore, the mechanism affected the curved portion of the dome only. Unfortunately, Boscovich report (Boscovich 1765) does not contain drawings that can be reproduced in this paper, but the description is clear enough to make understandable his ideas (a detailed interpretation of the mechanisms considered and of the one selected as the most detrimental can be found in Capecchi 2010b). The final

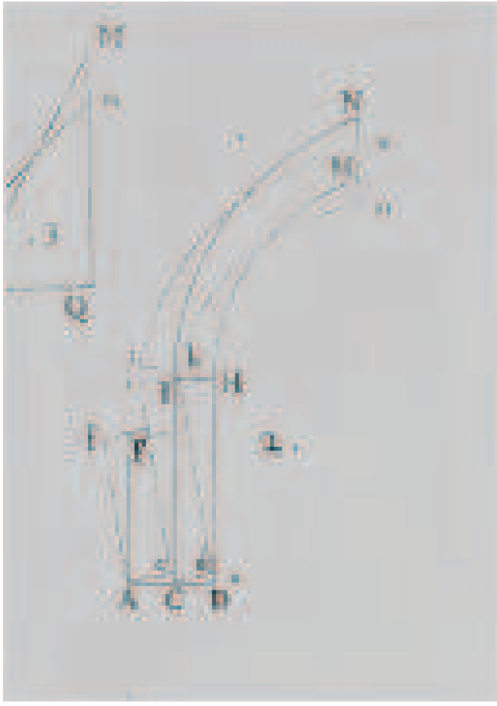


Fig. 7. The failure mechanism considered for St Peter's dome.

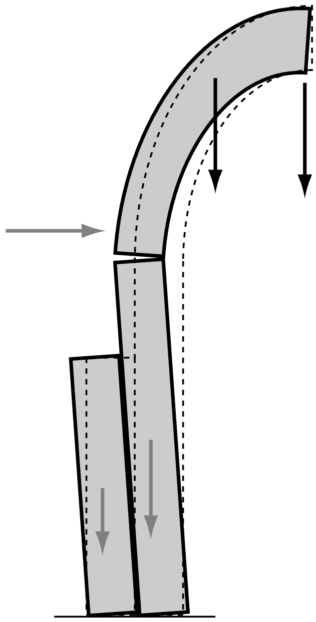


Fig. 8. Reversing (black) and stabilizing (gray) forces in St Peter's dome mechanism.

mechanism consisted of rigid blocks partly rotating and partly sliding with respect to each other, which is evidence of how Boscovich operated freely, not conditioned by preexisting schemes.

5. Remarks and comments

Some comments on Boscovich procedure and on its relation with the scientific developments of the time are in order. With respect to Belidor's method, the main difference is the extensive use of mechanisms based on mutual rotations between blocks. The shear type failure modality is replaced by a flexural one. In itself, this is not a novelty, since a similar scheme was previously proposed by Couplet, who introduced the term *hinge* to label the points where rotation occurred (Benvenuto 1981). Couplet mechanism, however, was fairly rigid, with the hinge position assigned *a priori*, regardless of the structural typology. Boscovich introduced considerable freedom, by making it an effective tool for the analysis of real cases.

The comparative merit of the flexural and shear type mechanisms are difficult to assess: the actual failure modality of a masonry structure is likely to involve both rotation and sliding of the different parts with respect to each other. Nevertheless, completely neglecting rotations appears too drastic and the flexural assumption also provides an operative benefit. At the time it was very nearly impossible to account for friction properly. Belidor neglected it, Boscovich in his study on Milan' cathedral considered some friction, but gave the corresponding force an arbitrary and rather unrealistic value. In a mechanism based on mutual sliding friction plays a crucial role, while one based on mutual rotations is almost insensitive to this aspect. Thus, an advantage of a flexural mechanisms is that results are to a large extent unaffected by a phenomenon which entails a considerable amount of uncertainty.

Boscovich procedure was not systematic. Each case had to be studied in its own and the failure modality was identified after examining and discarding a number of alternatives. In his work Boscovich showed much ingenuity and a great amount of what today is called *engineering judgment*, which is remarkable in a man whose main interests were of theoretical nature, but he did not set up a general methodology that other people could use effectively.

Actually, the times were ripe for such an operation. Before the end of the century Mascheroni, by refining some proposals advanced by Coulomb at about the same time, developed a general method for the analysis of masonry arches, vaults and domes, which was founded on mechanisms of flexural type, with the different parts of the structure mutually rotating about *hinges*. The location of hinges followed from overall equilibrium considerations and could be *computed* for each particular structural typology and loading case. The method encountered an immediate success and Mascheroni acquired a well deserved prestige.

Boscovich work anticipates this result, but very little credit is given to him in most histories of structural mechanics, which usually establish a direct link between Couplet and Mascheroni (see, e.g., Benvenuto 1981). Apparently, Boscovich work, focused as it was on specific problems, was not regarded as a contribution to the development of the subject and his reports attracted little attention among scientists (to whom, incidentally, were not directed).

One might wonder why Boscovich did not make any attempt at generalizing results that proved successful in two significant cases into an organized scheme that the scientific community of the time would recognize. After all, the ingredients of Mascheroni method were available to him and his capability was out of question. An obvious answer is that Boscovich acted to meet external requests and that neither pope Benedict XIV nor the *Veneranda Fabbrica del Duomo* asked for a general method. Certainly, he could have elaborated it by himself, by exploiting the experience gained when studying the two great cathedrals, but apparently he was not attracted by such an operation. His main interests were elsewhere and he abandoned the subject as soon as his commitments were accomplished.

6. Conclusions

In this paper attention was limited to the studies that Boscovich performed to assess the integrity and the load bearing capacity of masonry domes belonging to two monuments of notable significance, but most of the points raised and of the conclusions drawn apply to his entire work referring to applications of "engineering" character.

Boscovich did not tackle these problems spontaneously. It was his prestige and his well known reliability that brought them to him, since people that had to face difficult situations were keen to ask his advise. In all instances Boscovich produced fully satisfactory answers, showing a remarkable skill in dealing with practical problems. He got hold of the required data with accurate (for the time) measurements, examined different methods establishing their comparative merit, devised new strategies when available ones proved insufficient, went through detailed computations and discussed critically results pointing out their benefits and limitations. As a whole, he acted as an expert and very capable engineer, showing qualities that a man devoted to mathematics and astronomy does not possess necessarily.

Nevertheless, Boscovich involvement in these matters rarely proceeded beyond the accomplishment of his commitments. Seemengly he assumed that his mathematical knowledge and his own versatility allowed him to handle a wide range of problems, which, however, he did not consider of direct interest in themselves. His written reports were regarded essentially as private documents, addressed to the people who engaged his services, and this part of Boscovich production passed largely unnoticed to the scientific community, who gave it little credit.

Boscovich attitude perhaps explains this situation, which however hardly can be justified and severely underrates the scientific content of Boscovich "engineering" contributions. The reports on masonry domes, which were considered in this paper, not only represent perhaps the first instance in which the behavior of important structures was assessed by making use of sound mechanical concepts, but also contain a number of innovative ideas that demand attention by themselves. Though focused on rather specific problems and independently of the role they may, or may not, have played at the time in the development of the field, these reports are documents of significant value and must be regarded as outstanding scientific contributions.

References

Belidor, de B. F., 1734, La science des ingenieurs dans la conduite des travaux de fortification et d'architecture civile, Henry Scheurleer, La Haye.

Benvenuto E., 1981, La scienza delle costruzioni e il suo sviluppo storico, Sansoni, Firenze.

Boscovich R. G., 1765, Sentimento sulla solidità della nuova Guglia del Duomo di Milano, o si consideri in se stessa, o rispetto al rimanente del vasto Tempio, esposto a richiesta del Nobilissimo e Vigilantissimo Capitolo che sovraintende alla sua grande fabrica. Del Padre Ruggiero Giuseppe Boscovich della Compagnia di Gesù, in: Nava A., 1845, Relazione dei retauri intrapresi alla gran guglia del duomo di Milano nell'anno 1844 e ultimati nella primavera del corrente 1845, Valentini, Milano.

Capecchi D., 2010a, La perizia sulla cupola vaticana, in: Edizione Nazionale delle opere e della corrispondenza di Ruggiero Giuseppe Boscovich, vol. IX/II, pp. 11-86, Accademia Nazionale delle Scienze, Roma, ISBN 978 – 88 – 96700 – 06 – 8.

Capecchi D., 2010b, La perizia sul Duomo di Milano, in: Edizione Nazionale delle opere e della corrispondenza di Ruggiero Giuseppe Boscovich, vol. IX/II, pp. 209-246, Accademia Nazionale delle Scienze, Roma, ISBN 978 – 88 – 96700 – 06 – 8.

Le Seur T., Jacquier F., Boscovich G.R., 1742, Parere di tre mattematici sopra i danni che si trovano nella cupola di S. Pietro sul finire dell'anno MDCCXLII, dato per ordine di Nostro Signore Papa Benedetto XIV. Palearini, Roma.

Le Seur T., Jacquier F., Boscovich G.R., 1743, Riflessioni dei padri Tomaso Le Seur, Francesco Jacquier dell'Ordine dei Minimi e Ruggiero Boscovich della Compagnia di Gesù, sopra alcune difficoltà spettanti i danni, e risarcimenti della cupola di S. Pietro proposte nella congregazione tenutasi nel Quirinale a' 20 Gennaro MDCCXLIII e sopra alcune nuove ispezioni fatte dopo la medesima congregazione. Palearini, Roma.

Masotti A., 1962, Matematica e matematici nella storia di Milano, in: Storia di Milano, vol. XVI, pp. 713-814, Fondazione Treccani degli Alfieri, Milano.

Proverbio, E., ed., 2007, Catalogo delle Opere a Stampa di Ruggiero Giuseppe Boscovich (1711-1787), Accademia Nazionale delle Scienze, detta dei XL, ISSN 03 – 91 – 4666.

Stolfi G., 2003, La costruzione della guglia maggiore e il consulto dei matematici, in: ... E il Duomo toccò il cielo, E. Brivio and F. Repishti edts., pp. 165-176, Skira, Milano.