



${}^7\text{Li}$ abundance in pre-main sequence stars

Testing theory against clusters and binary systems

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Abstract. The disagreement between theory and observations for surface ${}^7\text{Li}$ abundance is a long standing problem which requires a quantitative analysis of the main uncertainties affecting theoretical models. We performed a detailed evaluation of the errors affecting ${}^7\text{Li}$ abundance predictions coming from the chemical composition and the adopted physical inputs. Then, present ${}^7\text{Li}$ abundances have been compared with the observations available for five young open clusters of different ages and chemical composition, namely, IC 2602, Per, Blanco1, Pleiades, and NGC 2516, and for four detached double-lined eclipsing binary systems. We confirm, for standard models, the disagreement between theoretical predictions and ${}^7\text{Li}$ observations for young open clusters if the same convection efficiency (mixing length parameter) during the pre-main sequence and the main sequence phase is adopted. However, if during the pre-main sequence a convection efficiency lower than the main sequence one is used, a satisfactory agreement with ${}^7\text{Li}$ observations can be achieved. Such value is with a good approximation independent of the age and the metallicity, at least in the range covered by our analysis.

Key words. Stars: abundances – evolution – interiors – low-mass – pre-main sequence – Hertzsprung-Russell and C-M diagrams

1. Introduction

Lithium is a fragile element, destroyed inside the stars via proton capture reactions at a relatively low temperature (about 2.5×10^6 K). Such temperatures can be easily reached in stars even during the early pre-main sequence (pre-MS) evolution. At this stage, the structure is almost fully convective, and thus it is mixed with the inner regions where nuclear burning is

much more efficient, producing an observable surface depletion of the most fragile elements, such as lithium. As the star evolves, the drop of the opacity in the inner regions, due to the temperature increase, leads to the formation of a radiative core. However, surface lithium depletion can continue in the convective envelope if the temperature is high enough to allow ${}^7\text{Li}$ burning. This makes surface lithium abundance strongly sensitive to the efficiency of the mixing processes in stellar envelopes.

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In the last two decades, a large amount of ${}^7\text{Li}$ data has been made available for open clusters, binary systems, and isolated stars (see e.g., Jeffries 2000; Sestito & Randich 2005, and references therein). Motivated by the improvements of the observational results, different theoretical models have been proposed through the years to explain the observed surface lithium abundance (see e.g. the reviews in, Charbonnel et al. 2000; Deliyannis et al. 2000; Pinsonneault et al. 2000). However, it is well known that model predictions disagree with the observations, even in the case of young pre-MS stars (see e.g., Jeffries 2000; D’Antona & Montalbán 2003; Montalbán & D’Antona 2006).

For sure, a crucial point in stellar computation concerns the treatment of the super-adiabatic convection efficiency. Generally, the *Mixing Length Theory* (Böhm-Vitense 1958) is adopted. This simplified scheme allows to evaluate the convection efficiency in those regions where large super-adiabatic gradient are required, as in the case of stellar envelopes, by introducing a free parameter to be calibrated. Commonly, it is adjusted to reproduce the solar radius. However, there are several hints (both theoretical and observational) about the possibility of a not constancy of the mixing length parameter, which can depend on the mass, chemical composition, and/or the evolutionary phase (see e.g., Chieffi et al. 1995; Ludwig et al. 1999; Morel et al. 2000; Ferraro et al. 2006; Trampedach 2007; Yıldız 2007; Gennaro et al. 2011; Piau et al. 2011).

We present a detailed discussion of the uncertainties on the predicted ${}^7\text{Li}$ surface abundance together with the results of the comparison between theoretical predictions and observation for surface ${}^7\text{Li}$ abundance in young open clusters and few pre-MS double-lined eclipsing binary systems. Present work is limited to stars younger than about 150 - 200 Myr, so that we can safely neglect the ${}^7\text{Li}$ depletion during the main sequence (MS) phase, which is caused by mechanisms operating on a time scale much longer than the pre-MS one (e.g. diffusion, radiative levitation, rotation). Indeed, during the pre-MS evolution surface ${}^7\text{Li}$ depletion is expected to be much more sen-

sitive to the convective mixing than to other non-standard mixing mechanisms. We also explore the effect of the convection efficiency on ${}^7\text{Li}$ surface abundance in the framework of the Mixing Length Theory, by adopting different values of the mixing length parameter during the pre-MS evolution.

2. Lithium data sample

${}^7\text{Li}$ data are taken from the large data sample made available by Sestito & Randich (2005), who performed an homogeneous re-analysis of surface lithium abundances in open clusters of several ages and chemical compositions ($5 \text{ Mr} \lesssim \text{age} \lesssim 8 \text{ Gyr}$, $-0.21 \lesssim [\text{Fe}/\text{H}] \lesssim +14$). Among the available clusters, we selected clusters younger than about 200 Myr, to avoid the effects of MS depletion (Sestito & Randich 2005), with a relatively high number of ${}^7\text{Li}$ data in a wide range of effective temperature, and different chemical compositions. The clusters that satisfy these criteria are Ic2602, α Per, Blanco1, Pleiades, and Ngc2516.

3. The models

Present models are computed by means of an up-to-date version of the FRANEC evolutionary code (Degl’Innocenti et al. 2008), as described in details in Tognelli et al. (2011). We briefly recall the main physical inputs adopted in this work. We adopt the updated version of OPAL EOS (Rogers & Nayfonov 2002) released in 2006, fully consistent with the OPAL (2005) high temperature radiative opacity¹ (Iglesias & Rogers 1996) for $\log T[\text{K}] > 4.5$ and the low-temperature radiative opacity by Ferguson et al. (2005) for $\log T[\text{K}] \leq 4.5$. Both the low- and high- temperature opacities are computed for the Asplund et al. (2005) solar mixture. Detailed boundary conditions are obtained from the Brott & Hauschildt (2005, BH05) atmospheric models for $T_{\text{eff}} \leq 10\,000 \text{ K}$, while for higher effective temperatures the Castelli & Kurucz (2003, CK03) atmosphere structures are adopted. The code follows the chemical evolution of the light elements (${}^2\text{H}$, ${}^6,{}^7\text{Li}$,

¹ The OPAL EOS and opacity tables are available at the url: <http://opalopacity.llnl.gov/>

${}^9\text{Be}$, ${}^{11}\text{B}$), whose reaction rates are taken from the NACRE compilation (Angulo et al. 1999). Microscopic diffusion is included in present computations, while radiative acceleration is not taken into account.

The logarithmic initial ${}^7\text{Li}$ content is set to $\epsilon_{\text{Li}} = 3.2 \pm 0.2^2$ (Jeffries 2006), representative of population I stars.

4. Uncertainties on predicted ${}^7\text{Li}$ abundance

In order to perform a robust comparison between present predictions and observed lithium abundance, it is mandatory to quantitatively analyse the uncertainties affecting the models. Stellar calculations rely on the adoption of several physical inputs (EOS, opacity, atmospheric models, nuclear reaction rates, convection efficiency), each characterized by its uncertainty. Moreover, the models requires the evaluation of the stellar chemical composition, which is affected by error too. The resulting predicted surface lithium abundance is consequently affected (at different levels) by all these uncertainty sources. Notice that, since surface lithium abundance is generally given as a function of the effective temperature of the star (T_{eff}), also the indetermination on T_{eff} due to such uncertainties must be evaluated.

We divide the uncertainty analysis into two sub-classes: the errors coming from the chemical composition and the ones due to the adopted physical inputs.

The helium abundance (Y) and the global metallicity (Z) are obtained from the spectroscopic $[\text{Fe}/\text{H}]$ values available in the literature by assuming a solar-scaled metal distribution, valid for population I star, and a commonly adopted linear relation between metals and helium abundance. Thus, Y and Z can be evaluated using the following relations,

$$Z = \frac{(1 - Y_{\text{p}})(Z/X)_{\odot}}{10^{-[\text{Fe}/\text{H}]} - (1 + \Delta Y/\Delta Z)(Z/X)_{\odot}} \quad (1)$$

$$Y = Y_{\text{p}} + Z \frac{\Delta Y}{\Delta Z} \quad (2)$$

² $\epsilon_{\text{Li}} = \log N_{\text{Li}}/N_{\text{H}} + 12$, where by N we denote the numerical abundance.

where Y_{p} is the primordial helium mass fraction abundance, $(Z/X)_{\odot}$ is the metals-to-hydrogen ratio as obtained from the analysis of the solar spectrum, and $\Delta Y/\Delta Z$ is the helium-to-metals enrichment ratio. For the present calculations we adopt the following reference values with their related uncertainties for these quantities: $Y_{\text{p}} = 0.2485 \pm 0.0005$ (Cyburt et al. 2004), $(Z/X)_{\odot} = 0.0181$ as obtained from the recent determination by Asplund et al. (2009) with an uncertainty of $\pm 15\%$ (Bahcall & Pinsonneault 2004), and $\Delta Y/\Delta Z = 2 \pm 1$ (Casagrande et al. 2007). Moreover, the measured $[\text{Fe}/\text{H}]$ value has its own error that generally lies in the range 0.01 - 0.1 dex. Thus, we adopt $\Delta[\text{Fe}/\text{H}] = \pm 0.05$ as a conservative uncertainty on $[\text{Fe}/\text{H}]$.

It is clear that uncertainties on each quantity appearing in eqs. (1) and (2) propagate into the uncertainty on Y and Z , and thus on predicted lithium abundance. We evaluated the error bars due to the indetermination on the chemical composition in the following way. We keep all the parameters of eqs. (1) and (2) constant to their reference value obtaining the reference values for (Y , Z). Then, we modify one by one the parameters by a maximum value given by its uncertainty (i.e. we modify the $[\text{Fe}/\text{H}]$ value by increasing or decreasing it by 0.05), obtaining new helium and metals abundances (Y' , Z'). The uncertainty on ϵ_{Li} and T_{eff} due to the input parameters is obtained by evaluating the difference between the sets of models with (Y , Z) and (Y' , Z'). Then the differences in ϵ_{Li} and T_{eff} evaluated for each parameter in eqs. (1) and (2) have been quadratically added to compute the final error bars on T_{eff} and ϵ_{Li} . We emphasize that this method is adopted to compute theoretical error bars for each cluster, hence for the $[\text{Fe}/\text{H}]$ value and age corresponding to each open cluster.

As an example, left panel of Fig. 1 shows the final error bars due to the uncertainty on the chemical composition for models with several stellar masses and ages, for $[\text{Fe}/\text{H}] = +0.0$.

Concerning the uncertainty coming from the adopted physical inputs, we focused our analysis on the effect of the radiative opacity coefficients, the EOS, and the lithium reaction rate. The total error coming from all these con-

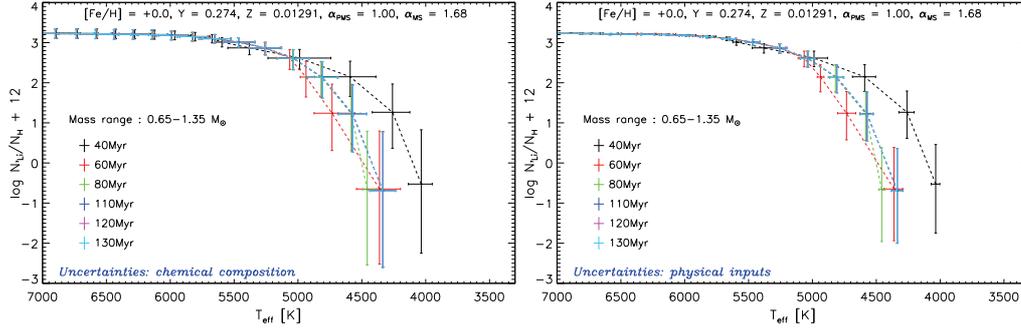


Fig. 1. Error bars on theoretical ${}^7\text{Li}$ surface abundance for several masses and ages, for the labelled chemical composition. The figure shows the contribution due to the uncertainty on the chemical composition (*left panel*) and on the adopted physical inputs (*right panel*).

tributions is shown in right panel of Fig. 1. The error bars have been computed by using exactly the same procedure described above for the chemical composition.

We adopt a conservative uncertainty of $\pm 5\%$ on the Rosseland mean opacity coefficients $\bar{\kappa}_R$ (see e.g., Neuforge-Verheeecke et al. 2001; Badnell et al. 2005). Notice that $\bar{\kappa}_R$ depends also on the assumed heavy elements abundances. However, this error source has already been taken into account in the evaluation of the uncertainties coming from the error on the chemical composition. The variation of $\bar{\kappa}_R$ has a strong effect on surface lithium abundance (about 1 - 2 dex for $M \lesssim 0.8 M_\odot$), since the opacity affects the temperature gradient and the maximum temperature reached at the bottom of the convective envelope, hence the rate of lithium destruction.

The uncertainty on the adopted EOS is evaluated by computing models with different equations of state widely used in the literature, namely the OPAL EOS 2006, 2001³, and the FreeEOS⁴ in the configuration that mimics the PTEH EOS (Polis et al. 1995). The EOS plays a crucial role in determining the temperature at the bottom of the convective envelope though

³ The OPAL EOS 2001 is available at the same url of the OPAL EOS 2006.

⁴ FreeEOS software and documentation are available at the url, <http://freeeos.sourceforge.net/documentation.html>

the adiabatic gradient and thus, indirectly, also the rate of lithium burning in this region.

We also analyse the effect of a variation of the lithium burning reaction rate of the order of the uncertainty on the astrophysical factor for this reaction, $\pm 5\%$ (see e.g., Lattuada et al. 2001). However, the effect on ϵ_{Li} is very small and completely negligible when compared to the other uncertainties. Moreover, we recall that a variation of the lithium burning rate does not modify the structure, and consequently T_{eff} is unaffected by such uncertainty.

5. Comparisons between theory and observations for surface ${}^7\text{Li}$ abundance in young open clusters and binary systems

We compute models with the chemical composition of five young open clusters, namely Ic2602, α Per, Blanco 1, Pleiades, and Ngc2516, as obtained from the most recent spectroscopic $[\text{Fe}/\text{H}]$ values (when available). The age and the best mixing length parameter for MS stars (α_{MS}) are determined from the fit of the colour-magnitude diagrams (CMD) with our theoretical isochrones. In this way the models adopted to calculate the surface lithium abundance are exactly the same used for the CMD fit.

The comparisons between our model predictions and clusters data are shown in Fig. 2. In the case of Ngc2516 we show the results

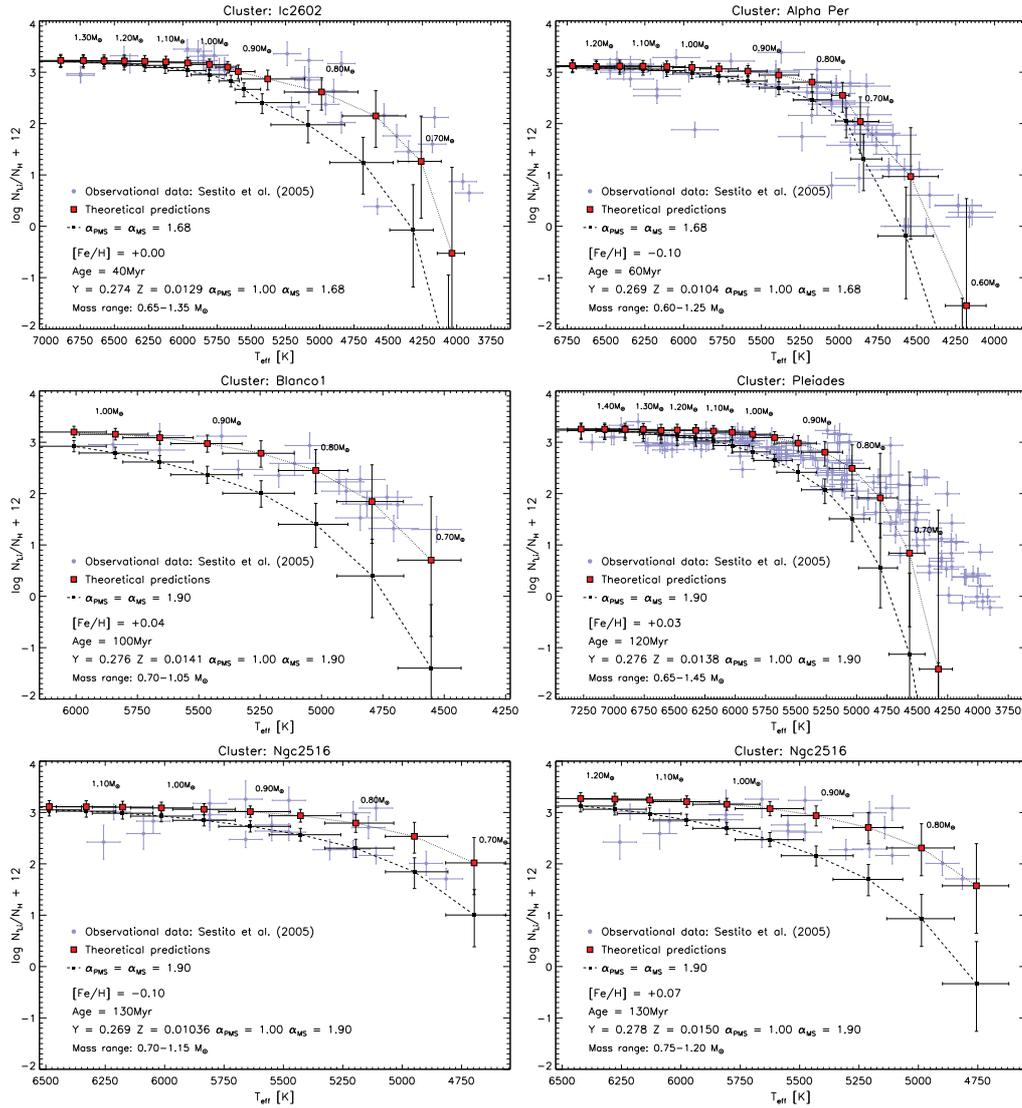


Fig. 2. Comparison between our theoretical models and surface ${}^7\text{Li}$ data for the five selected young open clusters. Each panel shows both the low-convection efficiency ($\alpha_{\text{PMS}} = 1.0$) and the high-convection efficiency models ($\alpha_{\text{PMS}} = \alpha_{\text{MS}}$).

for two different cluster metallicities, namely $[\text{Fe}/\text{H}] = -0.10$ (photometric determination, bottom left panel) and $[\text{Fe}/\text{H}] = +0.07$ (spectroscopic determination, bottom right panel) since the $[\text{Fe}/\text{H}]$ value for this cluster is affected by large uncertainties.

As a first step we adopted the value of the mixing length parameter calibrated on the CMD for MS stars, for all the evolutionary phases. If such value is adopted, the stars experience a strong pre-MS depletion, which is not compatible with the data, not even within the large theoretical error bars. Such discrep-

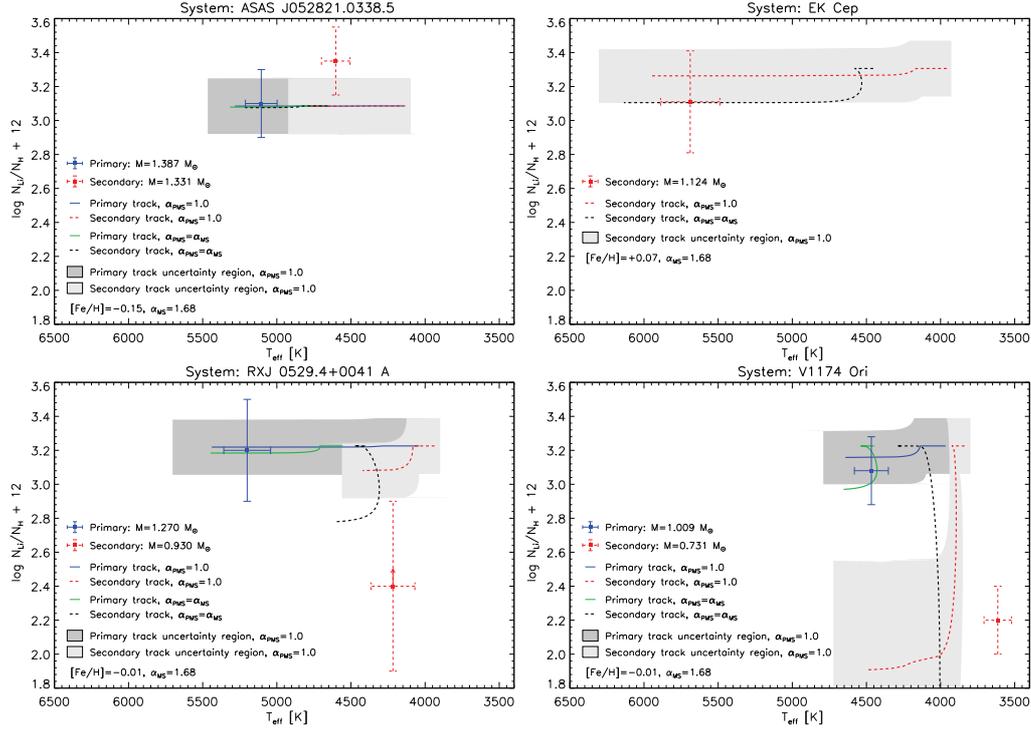


Fig. 3. Comparison between our theoretical models and surface ${}^7\text{Li}$ data in the selected pre-MS binary systems. ${}^7\text{Li}$ evolutionary tracks for the low-convection efficiency ($\alpha_{\text{PMS}}=1.0$) and the high-convection efficiency ($\alpha_{\text{PMS}} = 1.68$) are shown. The shaded areas represent the theoretical uncertainty regions, as discussed in Sect. 4.

any increases and gets worse as the stellar mass gets smaller, as it can be easily seen in Fig. 2 (black dashed line), confirming the well known large disagreement between predictions and data at least for standard models.

Given such a situation, we explore the possibility of a different convection efficiency during the pre-MS phase (α_{PMS}), computing models for different α_{PMS} values, namely $\alpha_{\text{PMS}} = 1.0, 1.2, 1.4, 1.68, 1.9$. For each cluster the best value of α_{PMS} is obtained by fitting the observed $\epsilon_{\text{Li}} - T_{\text{eff}}$ profile. In this way we can achieve a very good agreement between models and data if the value of $\alpha_{\text{PMS}} = 1.0$ is adopted in all the selected clusters, as shown in Fig. 2 (filled red squares and dotted line). Such value results to be independent on age, and $[\text{Fe}/\text{H}]$ value, at least for present clusters. The $\epsilon_{\text{Li}} - T_{\text{eff}}$ profile is well reproduced in all

the clusters, with the exception of the Pleiades, which do not show a satisfactory agreement with the data, although, due to the large uncertainty on the models, it is still compatible with the observations. Moreover, the Pleiades cluster shows a large dispersion in the surface lithium abundance among stars with similar effective temperature. Such a dispersion, which can be as large as 1 - 1.5 dex, makes the comparison more difficult and it could be the indication of additional lithium depletion mechanisms, or star-to-star differences in the original chemical composition. We also emphasize that the effective temperature of the selected clusters has been derived from the B-V color index, which could be altered by the presence of surface activity (see e.g. the discussion in, King et al. 2000; Xiong & Deng 2005). If this is the case, a shift in effective temperature is ex-

pected, and the observational uncertainties on T_{eff} could be larger than the quoted ones.

We perform a similar analysis also for four pre-MS detached double-lined eclipsing binaries (EBs), namely ASAS 052821A (Stempels et al. 2008), EK Cep (Popper 1987), RXJ+0529.4Aa (Covino et al. 2004), and V1174 Ori A (Stassun et al. 2004). For these objects the mass, luminosity, radius, effective temperature, and lithium abundance are given, hence we can compare the models with the data both in the HR diagram and in the $\epsilon_{\text{Li}} - T_{\text{eff}}$ plane. Among the four selected binary systems, ASAS, RXJ, and V1174 Ori have members close to their Hayashi track, a locus quite sensitive to the choice of α_{PMS} , while in the case of EK Cep both the components are close to the main sequence. Unfortunately, the uncertainty on T_{eff} for both the components of the selected systems are quite large when compared to the effect of a variation of the mixing length parameter, in a way that it is not easy to precisely constrain the best value of α_{PMS} . Nevertheless, we found low-convection efficiency models to agree reasonably well with the data in the HR diagram, especially in the case of the ASAS system. An exception is the V1174 Ori system, for which the models show the worst agreement with the data. However, this system seems to be peculiar in the sense that neither our present models nor the other models widely adopted in the literature are able to reproduce the position of the two companions simultaneously (see the discussion in Gennaro et al. 2011).

The comparison in $\epsilon_{\text{Li}} - T_{\text{eff}}$ plane are shown in Fig. 3. All the primary components of the selected systems experience a weak depletion during the pre-MS phase, due to their masses ($M_1 \gtrsim 1.0 M_{\odot}$), hence their surface lithium abundance is close to the initial one. On the contrary all the secondary components but ASAS system ($M_2 = 1.331 M_{\odot} \approx M_1$), undergo to a relevant pre-MS depletion. However, lithium data are affected by large uncertainties that avoid a precise determination of α_{PMS} . From the comparison one can only state that both the low- and high- convection efficiency models are compatible with the observations. Thus, contrarily to the case of young open clus-

ters, EBs do not exclude the possibility of an high-convection efficiency during the pre-MS phase, in the case of standard models

6. Conclusion

We performed a re-analysis of the disagreement between theoretical predictions and observation for surface ${}^7\text{Li}$ abundances, by comparing present theoretical models with ${}^7\text{Li}$ data available for young open clusters and pre-MS double-lined eclipsing binaries. In order to evaluate the level of the disagreement, we made a detailed analysis of the uncertainties affecting theoretical computations. The comparison between theory and observations for young open clusters showed that models with a low-convection efficiency during the pre-MS phase can satisfactorily reproduce the observed ${}^7\text{Li}$ abundances data in young open clusters. We performed a similar analysis also for four detached double-lined eclipsing binaries. In this case due to the large uncertainties on the data, it is not possible to firmly state the best α_{PMS} value. Indeed both the low- and high- convection efficiency models agree reasonably well with the data. These results point out that the treatment of the convection is a crucial issue in stellar modelling and that probably the simplified scheme of the mixing length theory is not completely adequate to describe this mechanism. Further improvements on the theoretical treatment of this phenomenon are thus mandatory.

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