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Towards a probabilistic approach for DAC and SAC exact reconstruction in hot emission stars

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Abstract. Spectral lines of hot emission stars show Discrete as well as Satellite Absorption Components (DACs and SACs). Disentangling the line components and identifying the physical mechanisms that produce them is a complex procedure, which is relying on many unknown parameters. The number of components that make up each line is one of the factors that need to be defined before examining the physical procedure producing the line profile. Here we propose a novel probabilistic approach to determine the exact number of line components as well as the statistical characteristics of each line component such as variance of the line or higher statistical moments that affect the shape of the DAC spectrum. This approach is based on the observation that the DAC lines may be treated as random signals which can be considered as superposition of independent signals. The number of independent signals can then be considered equal with the number of the interfered lines that make up the DAC. The determination of the higher statistical moments can be done based on the observation that each of the interfered independent random signals is Gaussian shaped signals in spectral space.

Key words. Stars: abundances – Stars: atmospheres – Stars: emission-line – Galaxy: globular clusters – Galaxy: abundances – Cosmology: observations

1. Introduction

The spectra of hot emission stars show lines with complex profiles in the UV region. This clearly indicates a multicomponent structure. In particular, they show absorption components shifted to the violet and/or to the red side of the main spectral line (Lyratzi et al. 2007). These components can either be discrete or blended. The lines are then characterized as Discrete Absorption Components (DACs) or Satellite Absorption Components (SACs) respectively. It is believed that the line components originate in different regions that have different radial and rotational velocities and a range of temperature and density. The lines can be produced either in the stellar photosphere or have a circumstellar or interstellar origin.

Based on the properties of the lines, researchers construct models to describe the properties and geometry of the physical system that produces the spectral lines. The exact number of line components, as well as their properties can reveal the physical mechanism that produces the lines and shed light into one

Send offprint requests to: M. Avlonitis of the most intriguing phenomena that take place in stars.

It is believed that DACs and SACs are the product of a series of complex physical processes. Models trying to reconstruct the phenomena that take place, involve independent density regions (such as blobs) with a range of properties. They are located around stars ejecting mass and/or envelopes around or near the star (Lyratzi et al. 2007). All models involve a range of phenomena such as stellar winds and magnetic fields (Cranmer et al. 2000; Danezis et al. 2003).

However, determining the number of components and their properties requires line fitting with a range of unknown factors using mathematical modeling applications. Therefore, the outcome of the fitting is constrained by confidence level. Here, we propose the use of a novel probabilistic method to reconstruct the DACs and SACs and extract the exact number of the components making up the spectral line as well as the statistical characteristics of each line component such as the line variance or higher statistical moments that affect the shape of the DAC spectrum. Hence, the unknown factors are reduced and the information can then be fed into a mathematical modeling application and return results with the highest possible confidence level. It is believed that such a method will refine the existing models and give as close as a complete picture of the geometry of the region as well as the physical mechanism that produces the lines.

In section 2, we summarize the basic aspects of the stochastic processes relevant to this work, while in section 3 we demonstrate this novel probabilistic approach for a three component line. Finally, in the last section a brief discussion of the limitations and potential of this method is presented.

2. Random field considerations

In this section we summarize some well known facts from stochastic processes. Indeed, for an arbitrary measured random signal it is possible to estimate its statistical properties by means of its statistical moments as well as its correlation and variance function. While the estimation of its statistical moments of any order is trivial the estimation of the correlation and variance function requires a more sophisticated procedure based on the notion of the moving averages (Vanmarcke 1983). In the next paragraph we focus mainly on the estimation of the variance function which closes the set of the equations of the proposed methodology in the next section. From a stationary random process $r(x)$ with mean $\bar{x}$ and variance s^2 a family of moving average processes $r_T(x)$ may be obtained as

$$r_T(x) = \frac{1}{T}\int_{x-T/2}^{r+T/2} r(x)dx, \qquad (1)$$

where T denotes the averaging window in space x. We define the variance function $\gamma(T) = s_T^2/s^2$ as the ratio of the variances of the resulting average patterns (after smoothing with average window T) over the original one. Then for a general class of processes the following relation holds (m is a pattern parameter and λ is the corresponding correlation length of the pattern)

$$\gamma(T) = \left[1 + (\frac{\lambda}{T})^m\right]^{-1/m}, \qquad (2)$$

or for fractal patterns (b is a parameter correlated to the fractal dimension)

$$\gamma(T) \propto T^{-b}, \qquad (3)$$

As a result given an arbitrary signal we can estimate signal data points $\gamma(T) = s_T^2/s^2$ vs

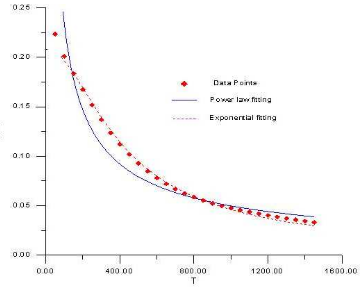

Fig. 1. The solid curve represents a power low behavior. It is evident that this does not fit correctly the data. In this case, exponential-like variance functions of the form of Eq. (3) is more appropriate.

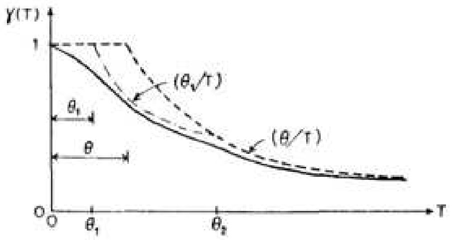

Fig. 2. The variance function of a two-component signal as well as the variance functions of the two component signals (θ corresponding to correlation lengths)

T using the above definitions. Plotting these points and finding the optimal fit of the functional type of Eq. (2) we may estimate the curve parameters λ, m or b . This procedure is depicted in Fig.1 for an arbitrary random signal.

Moreover in the case of composite random signals $r(x)$ defined as the sum of a number of independent component processes $r_i(x)$,

$$r(x) = r_1(x) + r_2(x) + ... = \sum r_i(x), \quad (4)$$

the following relations hold ([16]),

$$\bar{r} = \bar{r}_1 + \bar{r}_2 + ... = \sum \bar{r}_i, \quad (5)$$

and

$$s^2 = s_1^2 + s_2^2 + ... = \sum s_i^2, \quad (6)$$

The variance function of $r(x)$ is a weighted summation of component variance functions

$$\gamma(T) = \sum q_i \gamma_i(T), \quad (7)$$

the weights being the variance fractions q_i defined as,

$$q_i = \frac{s_i^2}{s^2}, \quad (8)$$

As an example, in Fig 2 the variance function of a two-component signal as well as the variance functions of the two component signals are depicted Vanmarcke (1983).

3. Proposed stochastic methodology for exact DACs reconstruction

In this section a novel probabilistic approach is proposed to address the problem of the exact reconstruction of DACs and SACs in hot emission stars. This involves the determination of the statistical properties up to second order each of the interfered line. The proposed probabilistic approach is based on the observation that the spectral lines may be interpret as random signals (the wavelength being the space variable) which can be considered as superposition of independent signals, the number of independent signals been equal with the number of the interfered lines that make up the DACs and SACs. In the following a three-component DAC line is considered in order to demonstrate the proposed methodology noting that generalization to more complex DAC lines is straightforward.

Let us assume a three-component DAC line, $r(x) = r_1(x) + r_2(x) + r_3(x)$ with $r_i(x)$ the interfered components. A set of six equation is needed in order to estimate the six unknown values, $\bar{r}_i, s_i^2, i = 1, 2, 3$. To this end we note that since $r_i(x)$ are uncorrelated the following relations hold,

$$\bar{r} = \bar{r}_1 + \bar{r}_2 + \bar{r}_3, \quad (9)$$

and

$$s^2 = s_1^2 + s_2^2 + s_3^2, \quad (10)$$

Moreover, the determination of the higher statistical moments can be done based on the observation that each of the interfered independent random signals is Gaussian-shaped signal in spectral space. As can be shown for a class of Gaussian-shaped signals there is a function relation between the first and second statistical moments (Appendix or forthcoming paper). As a result the following three equations are also true ($i = 1, 2, 3$),

$$s_i^2 = f_i(\bar{r}_i), \quad (11)$$

The missing equation in order to complete the system for the determination of the six unknown values, $\bar{r}_i, s_i^2, i = 1, 2, 3$ can be obtained

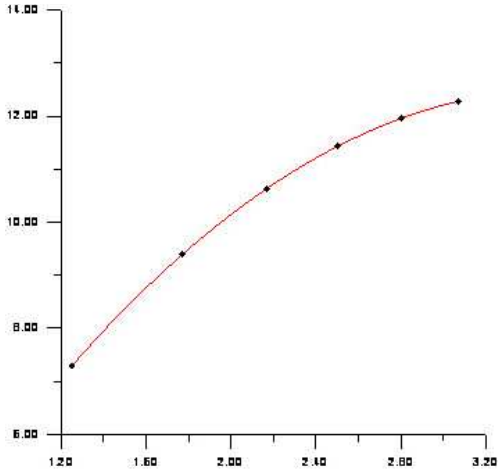

Fig. 3. The parabolic functional relation between first and second moment of Gaussian-shaped signals.

from the determination of the variance functions. Indeed, according to the previous section the following relation holds,

$$\gamma(T) = \sum \frac{s_i^2}{s^2}\gamma_i(T), \qquad (12)$$

The system of Eqs. (9) − (12) is our final result. Indeed, estimating $\bar{r}, s^2, \gamma(T), \gamma_i(T)$ by means of well known statistical methods (see discussion in the previous Section), the system of Eqs. (9) − (12) can be solved determined the six unknown values $\bar{r}_i, s_i^2, i = 1, 2, 3$.

4. Discussion

Summarizing the findings of the present paper we note that the proposed probabilistic approach can reconstruct information from spectral lines of hot emission stars giving with a mathematically rigorous manner the exact number of interfered spectral components as well as estimating with the highest possible confidence level the unknown parameters of the spectral line components which affect the shape of the monitored spectral line. It must be noted that the proposed formalism is based on the assumption of uncorrelated DACs and SACs spectral components. Relaxing this assumption, extra considerations must be made in order to reveal the functional relation between the second statistical moments of the correlated spectral components. Moreover the functional relation between first and second statistical moments must be studied for the general case of arbitrary Gaussian-shaped signals. We now plan to apply this new method to real data and extract the parameters of the lines.

5. Appendix

In order to find the relation between the first and second moment for the problem of DACs or SACs lines we created Gaussian-shaped signals with the same depth and arbitrary mean values and variances. Fig. 3 demonstrates the results for a wide range of testing parameters of arbitrary Gaussian-shaped signals. It can be seen that there is a one to one parabolic relation between the first and second moment.

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