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MHD theory of astrophysical jets

Kanaris Tsinganos

Department of Physics, University of Athens and IASA, Panepistimiopolis, 157 84 Zografos, Greece; e-mail: tsingan@phys.uoa.gr

Abstract. We review selected results of our analytical and numerical work on the construction of time-independent and time-dependent MHD models for non relativistic astrophysical outflows and jets. The 2-D MHD problem for steady and non steady axisymmetric magnetized and rotating plasma outflows is discussed and is shown that the only available exact solutions for such outflows are those in separable coordinates, i.e., those with the symmetry of radial or meridional self-similarity. Physically accepted solutions pass from the fast magnetosonic separatrix surface in order to satisfy MHD causality. An energetic criterion is outlined for selecting radially expanding winds from cylindrically expanding jets. Numerical simulations of magnetic self-collimation verify several results of analytical steady solutions. The outflow from solar-type inefficient magnetic rotators is very weakly collimated while that from a ten times faster rotating YSO produces a tightly collimated jet. We also propose a two-component model consisting of a wind outflow from the central object and a faster rotating outflow launched from the surrounding accretion disk which plays the role of the flow collimator. We also briefly discuss the problem of shock formation during the magnetic collimation of wind-type outflows into jets.

Key words. MHD – solar wind – ISM / Stars: jets and outflows – galaxies: jets

1. Introduction

A widespread phenomenon in astrophysics is the existence of collimated plasma outflows (jets) from the environment of several stellar or galactic objects (Tsinganos et al. 2009). The closer illustration of such highly collimated outflows are found in the relatively nearby regions of star formation. Thus, in the Orion Nebula alone the Hubble Space Telescope (HST) has observed hundreds of aligned Herbig-Haro objects (O' Dell & Wen 1994). In particular, HST observations show that several jets from young stars are highly collimated within about 30-50 AU from the source star with jet widths of the order of tens of AU, although their initial opening angle is rather large, e.g. $> 60^o$, (Bacciotti 2004; Ray 1996). There is also a long catalogue of jets associated with AGN and possibly supermassive black holes (Biretta 1996). To a less extend, jets are also associated with older mass losing stars and planetary nebulae (Livio 1997), symbiotic stars (Kafatos 1996), black hole X-ray transients (Mirabel & Rodriguez 1996), supersoft X-ray sources (Kahabka & Trumper 1996), low- and high-mass X-ray binaries and cataclysmic variables (Shahbaz et al. 1997). And, the most powerful sources in our Universe, gamma ray bursts, seem to be associated to the jet phenomenon and are understood as gamma

Send offprint requests to: K. Tsinganos ray synchrotron emission from ultrarelativistic jets.

In the theoretical front, the morphologies of collimated outflows have been studied, to a first approximation, in the framework of ideal steady or time-dependent magnetohydrodynamics (MHD). In *steady* studies, Blandford & Payne (1982) demonstrated that astrophysical jets may be accelerated magnetocentrifugally from Keplerian accretion disks, *if* the polodial fieldlines are inclined by an angle of 60^o, or less, to the disk midplane (but see also Cao 1997). This study introduced the often used "bead on a rotating rigid wire" picture, although these solutions are limited by the fact that they contain singularities along the system's axis and also terminate at finite heights above the disk, (Gracia et al. 2006; Vlahakis et al. 2000). The methodology of meridionally self-similar exact MHD solutions with a variable polytropic index was first introduced by Low & Tsinganos (1986) and Tsinganos & Low (1989) in an effort to model the *heated* axisymmetric solar wind. Heyvaerts & Norman (1989, 2003)) have shown analytically that the asymptotics of a particular fieldline in non isothermal polytropic outflows is parabolic if it does not enclose a net current to infinity; and, if a fieldline exists which does enclose a net current to infinity, then, somewhere in the flow there exists a cylindrically collimated core. Later, Bogovalov (1994) showed analytically that there *always* exists a fieldline in the outflowing part of a *rotating* magnetosphere which encloses a finite total poloidal current and therefore the asymptotics of the outflow *always* contains a cylindrically collimated core. In that connection, it has been shown in Bogovalov (1992) that the poloidal fieldlines are deflected towards the polar axis for the split monopole geometry and relativistic or nonrelativistic speeds of the outflowing plasma. Sauty & Tsinganos (1994) have self-consistently determined the shape of the fieldlines from the base of the outflow to infinity for nonpolytropic cases and provided a simple *criterion* for the transition of their asymptotical shape from conical (in *inefficient* magnetic rotators) to cylindrical (in *efficient* magnetic rotators). They have also conjectured that as a young star spins down losing angular momentum, its collimated jet-type outflow becomes gradually a conically expanding wind. Nevertheless, the degree of the collimation of the solar wind at *large* heliocentric distances remains still observationally unconfirmed, since spacecraft observations still offer ambiguous evidence on this question.

Another interesting property of collimated outflows has emerged from studies of various self-similar solutions, namely that in a large portion of them cylindrical collimation is obtained only after some oscillations of decaying amplitude in the jet-width appear (Vlahakis & Tsinganos 1997). In a series of papers, Sauty et al. (1999, 2002, 2004, 2005) have elaborated the physics of acceleration and collimation of MHD outflows by studying quasi-analytically *meridionally* self-similar outflows. Similarly, Vlahakis et al. (2000) have presented the first quasi-analytical example of a *radially* self-similar outflow which crosses the fast magnetosonic separatrix. The general properties of these MHD separatrices of the MHD equations have been discussed by Tsinganos et al. (1996). Radially self-similar models with cylindrical asymptotics for self-collimated and magnetically dominated outflows from accretion disks have been also constructed (Ostriker 1997). All existing cases of self-similar, jet- or, wind-type exact MHD solutions have been unified by Vlahakis & Tsinganos (1998) in a systematic analytical treatment wherein all available today examples of exact solutions emerge as special cases of a general formulation while at the same time new families with various asymptotical shapes, with (or without) oscillatory behavior emerge as a byproduct of this systematic method. Altogether, some general trends on the behavior of stationary, analytic, axisymmetric MHD solutions for MHD outflows seem to be well at hand.

However, observations seem to indicate that jets may inherently be variable. Thus, time-dependent simulations may be useful for a detailed comparison with the observations. Uchida & Shibata (1985) were the first to perform time-dependent simulations and demonstrate that a vertical disk magnetic field if twisted by the rotation of the disk can launch bipolar plasma ejections through the torsional Alfvén waves it generates. However, this mechanism applies to fully episodic plasma ejections and no final stationary state is reached to be compared with stationary studies. Similar numerical simulations of episodic outflows from Keplerian disks driven by torsional Alfvén waves on an initially vertical magnetic field have been presented in Ouyed & Pudritz (1997a,b). Goodson et al. (1997) have proposed a time-dependent jet launching and collimating mechanism which produces a two-component outflow: hot, well collimated jet around the rotation axis and a cool but slower disk-wind. Bogovalov (1996, 1997) modeled numerically the effects of the Lorentz force in accelerating and collimating a cold plasma with an initially monopole-type magnetic field, in a region limited also by computer time, i.e., the near zone to the central spherical object. In a series of papers, Bogovalov & Tsinganos (1999, 2001, 2005); Tsinganos & Bogovalov (2000, 2002) have used a novel way to model numerically MHD outflows to very large distances by first integrating the time-dependent MHD equations in the near zone containing the critical surfaces and then extending these solutions to unlimited distances along the jet in the superfast regime. In this article we shall constrain our attention only on jets and not to their associated accretion disks. In Lebedev et al. (2005); Ciardi et al. (2006) the interested reader may find more on laboratory plasma jets and magnetic tower outflows from a radial wire array in the Z-pinch. We do not intend to discuss instabilities in jets and the interested reader may see, e.g., Kersale et al. (2000) for pressure and magnetic shear-driven instabilities and Trussoni (2007) for Kelvin-Helmholtz instabilities in rotating MHD jets.

The paper is organized as follows. After the Introduction, Secs. 2,3 deal with 2-D steady MHD outflows and in particular the various classes of self-similar solutions, the nature of the MHD separatrices selecting a physically acceptable solution and the criteria for collimation. Secs. 4,5 present results on numerical simulations of MHD outflows and gives a simple example to explain why magnetized and rotating jets are collimated. Sec. 6 outlines our two-component model which we believe describes appropriately cosmical jets, in particular those associated with YSO's.

2. Steady-state, exact self-similar MHD outflows

Magnetized outflows are described (to first order) by the ideal MHD equations, i.e., Maxwell's equations combined with the conservation of mass, momentum, and energy,

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = 4\pi\delta \approx 0\,,\quad \boldsymbol{\nabla}\times\boldsymbol{E} = -\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}\,, \tag{1}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0\,,\quad \boldsymbol{\nabla}\times\boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t} \simeq \frac{4\pi}{c}\boldsymbol{J}\,, \tag{2}$$

coupled to Ohm's law for a plasma of very high electrical conductivity σ,

$$\boldsymbol{E} + \frac{\boldsymbol{V}\times\boldsymbol{B}}{c} = \frac{\boldsymbol{J}}{\sigma} \simeq 0\,, \tag{3}$$

Newton's law expressing conservation of angular momentum,

$$\rho\frac{\partial\boldsymbol{V}}{\partial t} + (\rho\boldsymbol{V}\cdot\boldsymbol{\nabla})\boldsymbol{V} = -\boldsymbol{\nabla}P + \frac{\boldsymbol{J}\times\boldsymbol{B}}{c} - \rho\boldsymbol{G}\,, \tag{4}$$

the continuity equation expressing conservation of mass,

$$\boldsymbol{\nabla}\cdot\rho\boldsymbol{V} + \frac{\partial\rho}{\partial t} = 0\,, \tag{5}$$

and finally an equation for energy conservation,

$$\rho\left[P\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\rho}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P}{\rho(\Gamma-1)}\right)\right] = \\ = \rho\frac{\partial h}{\partial t} - \frac{\partial P}{\partial t} + \rho\boldsymbol{V}\cdot\left[\nabla h - \frac{\nabla P}{\rho}\right] = q\,. \tag{6}$$

The symbols have their usual meaning, i.e., $\boldsymbol{V}(x_1,x_2,x_3,t)$, $\boldsymbol{B}(x_1,x_2,x_3,t)$ are the bulk flow speed and magnetic field in the plasma which is generated by an electric current with surface density $\boldsymbol{J}(x_1,x_2,x_3,t)$, $\boldsymbol{G}(x_1,x_2,x_3)$ is the external gravitational field, $\rho(x_1,x_2,x_3,t)$ and

$P(x_1, x_2, x_3, t)$ are the plasma density and pressure, $h(x_1, x_2, x_3, t)$ the enthalpy of the gas [= $(\Gamma/\Gamma - 1)(P/\rho)$] and finally $q(x_1, x_2, x_3, t)$ denotes the volumetric rate of energy addition, all expressed in some curvilinear coordinates (x_1, x_2, x_3).

Assuming steady-state and axisymmetry, several conserved quantities along the flow exist (Tsinganos 1982). If we label each poloidal field line with the poloidal magnetic flux function A, they are the mass-to-magnetic flux ratio, $\Psi_A(A)$,

$$A = \frac{1}{2\pi}\int \mathbf{B}_p \cdot d\mathbf{S}\,, \quad \Psi_A(A) = \frac{4\pi\rho V_p}{B_p}\,, \tag{7}$$

the field angular velocity $\Omega(A)$ and the specific total angular momentum $L(A)$

$$\Omega(A) = (V_\varphi/\varpi) - (V_p/\varpi)(B_\varphi/B_p)\,, \tag{8}$$

$$L(A) = \varpi V_\varphi - \varpi B_\varphi/\Psi_A\,, \tag{9}$$

while the Alfvénic lever arm ϖ_a on each field line is, $\varpi_a = \sqrt{L(A)/\Omega(A)}$ (Tsinganos 1982). It is convenient to introduce two more dimensionless functions, the Alfvén Mach number M and the cylindrical distance in units of ϖ_a,

$$M = \frac{V_p}{B_p/\sqrt{4\pi\rho}}\,, \quad G = \frac{\varpi}{\varpi_a}\,. \tag{10}$$

All physical quantities can be expressed as functions of the magnetic flux function A [i.e., $\varpi_a(A)$, $\Psi_A(A)$, $\Omega(A)$] and the two variables (G, M) (e.g. see Tsinganos 1982).

$$\mathbf{B} = \frac{dA}{d\varpi_a}\nabla\frac{\varpi}{G} \times \frac{\hat{\varphi}}{\varpi} - \frac{\varpi_a^2\Omega\Psi_A}{\varpi}\frac{1-G^2}{1-M^2}\hat{\varphi}\,, \tag{11}$$

$$\mathbf{V} = \frac{M^2}{\Psi_A}\mathbf{B}_p + \frac{\varpi_a^2\Omega}{\varpi}\frac{G^2-M^2}{1-M^2}\hat{\varphi}\,. \tag{12}$$

The functions $G(r, \theta)$ and $M(r, \theta)$ can be obtained by integrating the two coupled components of the momentum equation on the poloidal plane. However, due to the complexity of these two partial differential equations (PDE) and in order to proceed semi-analytically, we are forced to make further assumptions. For example, if we are interested for a nonlinear separation of the variables in the two PDE, we may employ the only known at present approach, namely the method of self-similarity.

Fig. 1. A characteristic example of Russian matryoshka

A physical phenomenon is called *temporarily* self-similar if it can be reproduced at any time via a self-similar mechanism from a previous temporal state. The classical such example is a nuclear bomb explosion with the mushroom typical shape. Analogously, a physical phenomenon is called *spatially* self-similar if it can be reproduced everywhere in space via a spatial self-similar mechanism. The classical such example are the Russian matryoshkas, Fig. 1. Also, the observed shapes of astrophysical jets in the galactic and extragalactic scales, are suggestive of such a symmetry in space. Technically, spatial self-similarity may be viewed as a method of nonlinear separation of the variables in the set of the steady MHD equations, providing us the opportunity to obtain analytically solutions.

In particular, in self-similarity it is assumed that both G and M are functions of a single variable χ (Vlahakis & Tsinganos 1998). If this is the case, the ratios $(1 - G^2)/(1 - M^2)$, $(G^2 - M^2)/(1 - M^2)$ appearing in Eqs. (11) and (12) are functions of χ only, and the components of the momentum equation become relatively simple expressions of χ and ϖ_a. It is in principle possible to choose the functional form of the integrals such that the variables (χ, ϖ_a) decouple, in which case the equations become ordinary differential (ODEs) with re-

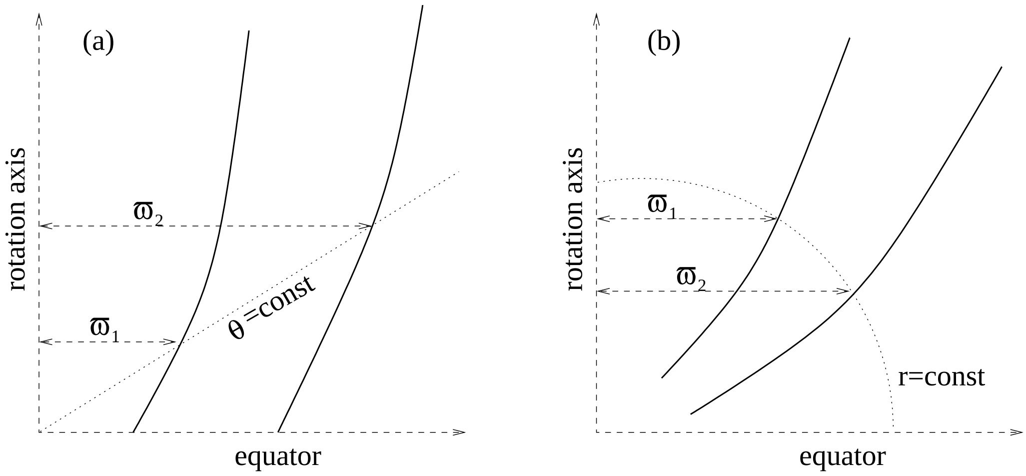

Fig. 2. An illustration of the two possibilities of self-similar field line structure. Consider two arbitrary field lines (thick lines). In (*a*) is illustrated the case of radial self-similarity, wherein the ratio ϖ_1/ϖ_2 for the intersection of any poloidal line with a cone is the same for any value of θ. Specifically, if we write $A(r,\theta) = \varpi^x f(\theta)$, the ratio of the cylindrical distances ϖ_1 and ϖ_2 in which the direction θ=const. intersects the two lines $A = A_1$ and $A = A_2$ is $A_1/A_2 = (\varpi_1/\varpi_2)^x$. Thus, if the MHD problem is solved for one fieldline $A = A_1$, i.e., the unknown function $f(\theta)$ is calculated, from the distance ϖ_1 we may calculate ϖ_2, $\varpi_2 = A_2/A_1{}^{1/x}\varpi_1$ for given values of $A = A_1$ and $A = A_2$. Thus, if we know one field line we may construct all the others. In (*b*) it is illustrated the case of meridional self-similarity. By writing $A(r,\theta) = \varpi^x f(r)$, a determination of the unknown function $f(r)$ yields the ratio of the cylindrical distances ϖ_1 and ϖ_2 in which the sphere r=const. intersects the two lines $A = A_1$ and $A = A_2$. This ratio is the same for any spherical surface r =const., i.e., $\varpi_2 = (A_2/A_1)^{1/2}\varpi_1$. Thus, again if we know one field line we may construct all the others.

spect to χ. The only remaining difficulty then is that the solution should cross various singular points, corresponding to ratios $\frac{0}{0}$ in the ODEs.

This unifying scheme contains two large families of exact MHD models, which are systematically constructed in Vlahakis & Tsinganos (1998):

1. For $\chi = \theta$ we get the family of the *radially* self-similar models with conical critical surfaces and with prototype the Blandford & Payne (1982) model, (see also Vlahakis & Königl 2003, for the relativistic case). Figure 2(a) illustrates the radial self-similar character of the poloidal field lines, resulting from the assumption $\varpi = \varpi_a G(\theta)$.
2. For $\chi = r$ we get the family of the *meridionally* self-similar models with spherical critical surfaces and with prototype the Sauty & Tsinganos (1994) model (henceforth ST94 model (see also Sauty et al. 2004). This family also includes the classical Parker (1963) description of a stellar wind, as its simplest member; it also contains the simple prescribed field line model of Tsinganos & Trussoni (1991). Figure 2(b) illustrates the meaning of the meridional self-similar assumption $\varpi = \varpi_a G(r)$.

It is worth to note that in the class of meridionally self-similar solutions analyzed in Sauty & Tsinganos (1994) an interesting parametric energetic criterion emerges which characterizes the asymptotic shape of the streamlines. In terms of this parameter, we may either have an Efficient Magnetic Rotator (**EMR**) to magnetically collimate the outflow into a jet,

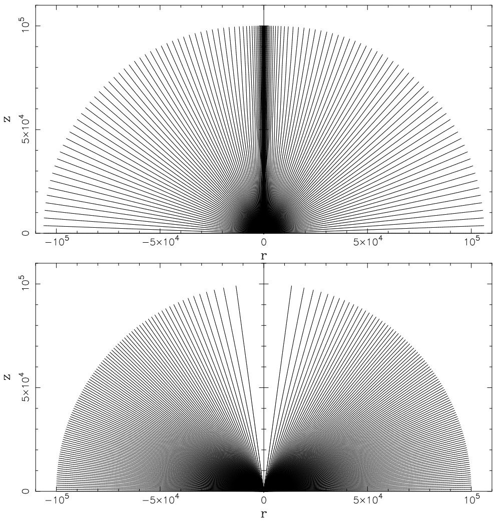

Fig. 3. In the top panel the poloidal magnetic field lines of the rotating outflow are plotted in the far zone and for intervals of equal magnetic flux $\Delta\Phi = 10^{-2}$ for a total normalized flux $\Phi = 1$. For comparison, the original (t=0) nonrotating and uncollimated initial monopole magnetosphere is shown in the bottom panel.

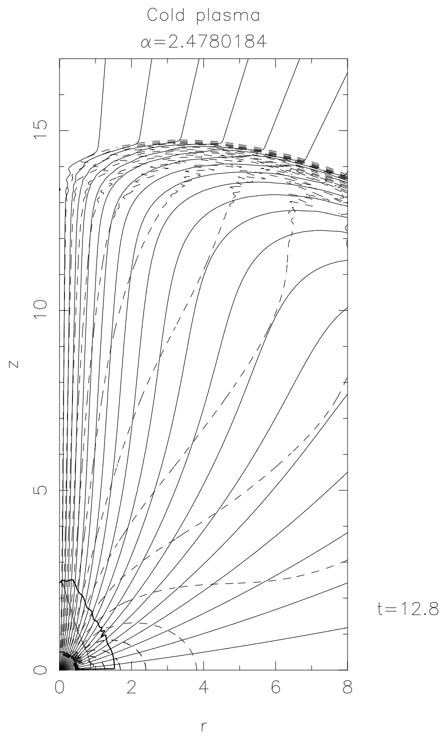

Fig. 4. Intermediate state, wherein after the beginning of rotation of the base of the outflow, the magnetic field lines focus towards the axis of rotation, by the Lorentz $\boldsymbol{j} \times \boldsymbol{B}$ force. An MHD shock wave propagates downstream the information of the rotation and collimates at large distances the outflow.

or, an Inefficient Magnetic Rotator (**IMR**. This EMR/IMR criterion is an extension to 2-D of the FMR/SMR criterion in the 1-D Weber & Davis model.

3. Critical points, separatrices and causality in MHD flows

An interesting feature of axisymmetric MHD wind-type solutions is the appearance of two X-type critical points within the flow domain, in addition to the Alfvén critical point. In general, at the critical points the bulk flow speed equals to one of the characteristic speeds in the problem. Hence, it is of physical interest to associate the flow speeds at these X-type critical points to a characteristic speed for MHD wave propagation. In that connection, first note that these semi-analytical solutions possess the symmetries of self-similarity and axial symmetry. Thus, in spherical coordinates (r, θ, φ), the self-similarity direction $\hat{s}$ can be $\hat{s} \equiv \hat{\theta}$, or, $\hat{s} \equiv \hat{r}$ and the axisymmetry direction is $\hat{\varphi}$. Therefore, a wave that preserves those two symmetries should propagate along the $\hat{\varphi} \times \hat{s} \parallel \hat{\chi}$-direction in the meridional plane. Besides the incompressible Alfvén mode propagating along the magnetic field (**B**) with velocity V_A, the compressible slow/fast MHD modes propagate in the direction $\hat{\chi}$ with phase speeds $V_\chi \equiv V_{slow,\chi}$, or, $V_\chi \equiv V_{fast,\chi}$ which satisfy the quartic

$$V_\chi^4 - V_\chi^2(V_A^2 + C_s^2) + C_s^2 V_{A,\chi}^2 = 0\,. \qquad (13)$$

Hence, when the above equation is satisfied the governing equations have X-type singularities and $V_\chi = \mathbf{V} \cdot \hat{\chi}$.

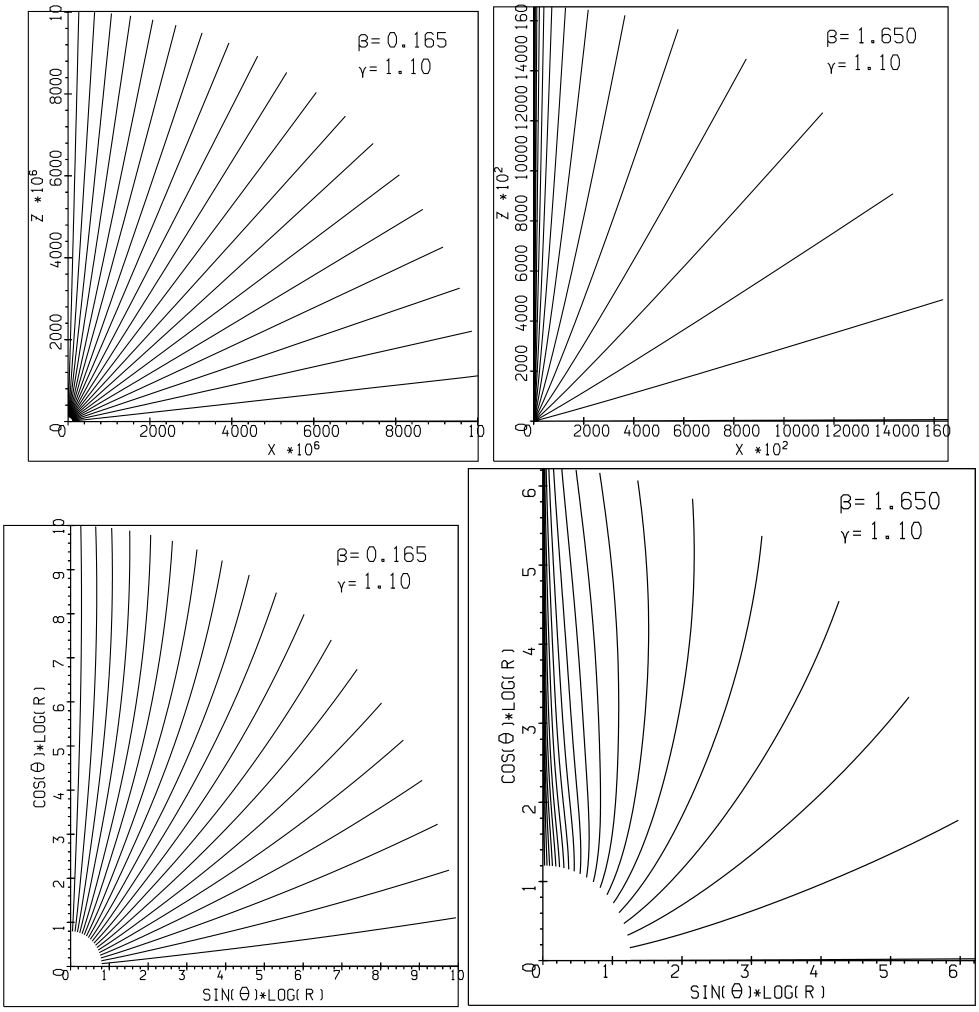

Fig. 5. *Left panels*: Shape of poloidal magnetic field lines in the far zone of an assumed isotropic solar wind. In the upper panel the poloidal field lines are plotted in a linear scale, while in the lower panel in a logarithmic scale which magnifies their slight bending towards the axis. As it may be seen, collimation is negligible. *Right panels*: Same as in the left panels but for a wind from a star rotating 10 times faster than the Sun. Note the significant collimation in this case.

On the other hand, it is well known that in the MHD flow system there exist two hyperbolic regimes wherein characteristics exist: the inner hyperbolic regime which is bounded by the cusp and the slow magnetosonic surfaces and the outer hyperbolic regime extending downstream of the fast magnetosonic point. Within each of those two hyperbolic regimes, there exists one limiting characteristic or separatrix surface: the slow magnetoacoustic separatrix surface (SMSS) inside the inner hyperbolic regime and the fast magnetoacoustic separatrix surface (FMSS) inside the outer hyperbolic regime (Bogovalov 1994; Tsinganos et al. 1996). The true critical points are found precisely on these two separatrices. Furthermore, the FMSS plays the role of the MHD signal horizon of the problem in the

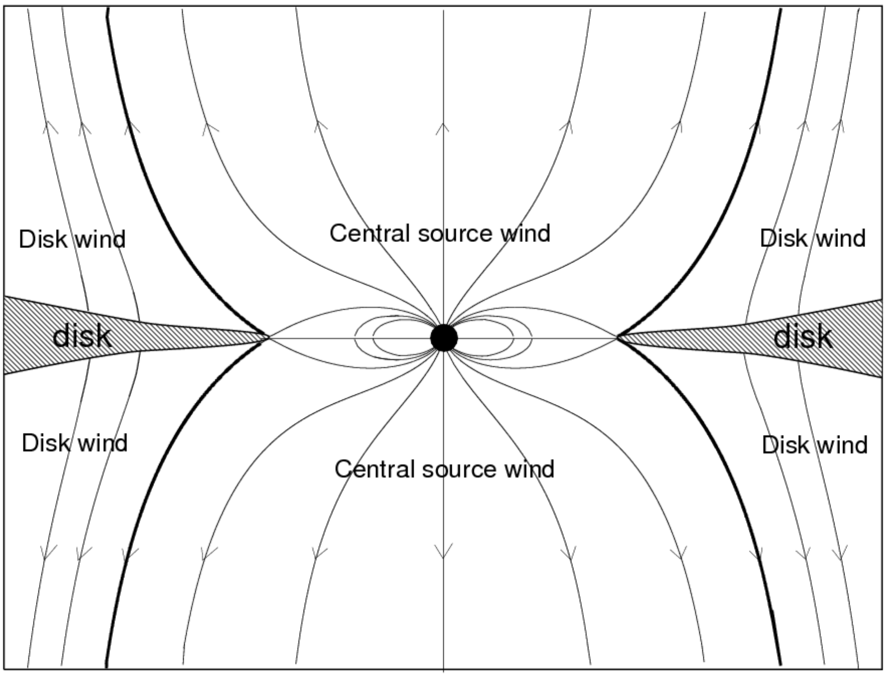

Fig. 6. A theoretician's illustration of the two-component model. A central source emits an initially roughly radially expanding at the base outflow. This stellar wind is self-collimated if the protostar is an EMR. If the protostar is an IMR, the stellar wind is assisted to collimate by the disk wind from the rapidly rotating inner edges of the surrounding accretion disk which is easily self-collimated. Arrows indicate the magnetic field. Typical dimensions for the system of a YSO are : protostellar radius, $R_* = 3R_\odot$, disk inner radius (magnetospheric cavity) $\varpi_i = 0.1AU$, jet emitting part of the disk, 2.5 AU, disk outer radius $\varpi_e = 100AU$. The jet carries most of the angular momentum of the accreting gas which then falls on the protostar along the paths of the magnetic field in the magnetospheric cavity.

sense that if the poloidal outflow speed exceeds the corresponding speed at the FMSS, then no perturbation can affect the solution upstream of the FMSS. In other words, setting the boundary conditions at the FMSS is a proxy of setting the correct boundary conditions at infinity. As two examples, in a meridionally self-similar case analysed in Sauty et al. (2004), the SMSS is at $R = 0.751$ while the FMSS is located at $R = 4150$, while an analogous situation for radially self-similar cases with two critical transitions at the SMSS ($M_{ms} = 1$) and FMSS ($M_{mf} = 1$)] is shown in Vlahakis et al. (2000).

4. Numerical simulations of jet formation

An investigation of the problem of the collimation of a MHD outflow can be also obtained through a numerical simulation of the time-dependent MHD equations. In one approach we have employed, the simulation can be done in two steps. *First*, a steady state solution in the nearest zone which contains the relevant MHD critical surfaces and the governing PDE are of mixed elliptic/hyperbolic type is obtained by using a relaxation method (Bogovalov & Tsinganos 2001; Tsinganos & Bogovalov 2002). In the *second* step, the solution in the far zone can be obtained by extending to large distances the solution obtained in the nearest zone. This ability to extend the inner zone solution is based on the fact that the outflow in the far zone is already superfast magnetosonic. Therefore, the problem can be treated as an initial value Cauchy-type problem with the initial values taken from the solution of the problem in the nearest zone. The advantage of this method is that large lengths of the jet can be modeled. A second method employs direct time-dependent numerical simulation from the base of the outflow, but in this way inevitably a smaller length of the jet is modeled (c.f. Casse 2004; Krasnopolsky et al. 2000; Ouyed & Pudritz 1997a; Ustyugova et al. 1999; Zanni et al. 2004).

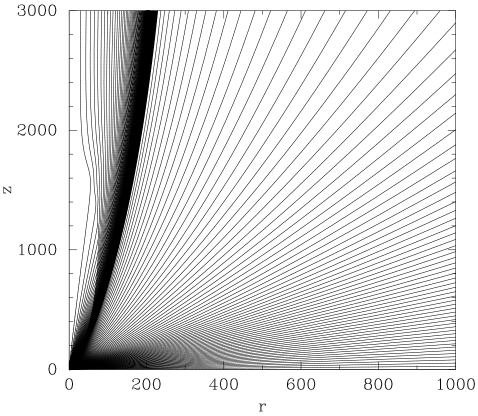

Fig. 7. The result of the simulation for the configuration shown in the previous Fig. 6 (Bogovalov & Tsinganos 2005). When the disk starts rotating its disk wind collimates forcing the inner wind to collimation too. A byproduct of the collision of the two outflow components is the formation of a shock at the interface of the central wind and the surrounding disk-wind.

5. Why is a magnetized and rotating outflow collimated?

To illustrate in simple terms the effects of rotation and magnetic fields in the outflow of a plasma from a central gravitating object, consider a monopole-type magnetic field, $B_r = B_o/R^2$, where B_o is the magnetic field at the base $R = r/r_o = 1$. Assume that the plasma flows with a constant speed V_o along these radial magnetic field lines. The Alfvén number of this outflow is,

$$M^2(R) = \frac{V_0^2}{V_A^2} = 4\pi\rho\frac{V_0^2}{B_0^2}R^4 = 4\pi\rho_a\frac{V_0^2}{B_0^2}\frac{R^2}{R_a^2}\,. \quad (14)$$

where from mass conservation $\rho = \rho_a(R_a^2/R^2)$. Since at the Alfvén radial distance $R_a : M = 1$, $4\pi\rho_a V_0^2 = B_0^2$ we have finally,

$$M = \frac{R}{R_a}\,. \quad (15)$$

Let us assume that the base of the outflow rotates with an angular velocity Ω. From the steady MHD equations the induced azimuthal magnetic field B_φ is

$$\frac{B_\varphi}{B_r} = -\frac{\Omega\varpi_a^2}{\varpi(B_r/\Psi_A)}\frac{\varpi^2/\varpi_a^2 - 1}{M^2 - 1}\,, \quad (16)$$

where Ψ_A is the mass flux per unit of magnetic flux. Let us assume for the moment that the outflow remains radial after the rotation starts. Then, $\varpi = R\sin\theta$ such that $M = \varpi/\varpi_a$ and for distances much larger than the Alfvénic cylindrical distance, $\varpi \gg \varpi_a$, $R \gg R_a$, $M^2 \approx \varpi^2/\varpi_a^2$, it follows that

$$\frac{B_\varphi}{B_r} \approx -\frac{\Omega\varpi_a^2}{\varpi}\frac{\Psi_A}{B_r} = -\frac{\Omega\varpi_a^2}{\varpi}\frac{\Psi_A^2}{4\pi\rho}\frac{4\pi\rho}{\Psi_A B_r} =$$

$$= -\frac{\Omega\varpi_a^2}{\varpi}\frac{\Psi_A^2}{4\pi\rho}\frac{1}{V_0} \quad (17)$$

But,

$$\frac{\Psi_A^2}{4\pi\rho} = \frac{4\pi\rho V_0^2}{B_r^2} = M^2 \approx \frac{\varpi^2}{\varpi_a^2}\,, \quad (18)$$

and thus,

$$\frac{B_\varphi}{B_r} \approx -\frac{\Omega}{V_0}\varpi\,. \quad (19)$$

i.e., the azimuthal magnetic field grows with the cylindrical distance ϖ in relation to the poloidal magnetic field B_r. Thus, although at the rotation axis the magnetic tension is negligible, the azimuthal magnetic field grows with distance from the axis of rotation and eventually it will dominate over the poloidal magnetic field B_p. The magnetic pressure and tension then exert a net force towards the axis of rotation and one may wonder for what might balance this inwards force. The outward inertial (centrifugal) force $\rho V_\varphi^2/\varpi$ is negligible since the azimuthal flow speed is negligible in the same approximation,

$$V_\varphi = \frac{\Omega\varpi_a^2}{\varpi}\left[1 - \frac{\varpi^2/\varpi_a^2 - 1}{M^2 - 1}\right] \approx 0\,. \tag{20}$$

The last available means to balance the inwards hoop stress would be some suitable pressure gradient. However, the magnetic pressure drops with the cylindrical distance ϖ like $1/\varpi^2$ and is negligible. The thermal gas pressure on the other hand, should drop like $1/\varpi^3$ in an atmosphere where $V = V_o$ ($\rho \sim \varpi^{-2}$) in order that the thermal pressure gradient balances gravity. It follows that the unavoidable result is that magnetic tension will bend the poloidal magnetic field lines towards the axis, forming a cylindrical core. Such a dramatic formation of an inner jet by magnetic self-collimation may be seen in Fig. 3(a) after we start rotating the initial radial magnetosphere of Fig. 3(b). In Fig. 5 we rediscover the result of the steady MHD modeling that a fast magnetic rotator (in this case a YSO rotating 10 times faster than the Sun, produces a tightly collimated jet while the solar wind does not show any significant collimation.

6. A two-component model for jets from the system of a central source and a disk

A serious limitation however of the previous simulations of magnetic collimation is that only a tiny fraction of order $\sim$ 1% of the mass and magnetic flux of the originally radial wind ends up collimated inside the jet (Bogovalov & Tsinganos 2001). Similarly, in analytical models if the source of the wind is a stellar surface and the disk does not feed the outflow with mass and magnetic flux, very low wind- and jet-mass loss rates ($\dot{M}_{wind}$, $\dot{M}_j$) are obtained. However, in outflows associated with YSO current estimates place $\dot{M}_{jet}$ in the limits $\dot{M}_{jet} \sim 10^{-6} - 10^{-8} M_\odot/yr$ (Ray 1996). And, the inferred from observations mass loss rates of bipolar outflows indicate wind mass loss rates also in the range of $\dot{M}_{wind} \sim 10^{-6} - 10^{-8} M_\odot/yr$, depending largely on the luminosity of the YSO's. Therefore, the mass loss rate in the jet has to be a large fraction of the mass loss rate in the surrounding wind. The idea that the source of the jet rotates rather slowly may be quite reasonable, at least in relation to YSO's. It is evident that a protostar should rotate more slowly than the inner edges of its Keplerian accretion disk and observations indeed confirm this prediction (Bacciotti 2004). We do not intend to argue here that the matter in the jet is ejected from the protostar. The close disk-jet connection (Hartigan et al. 1995) shows that the matter in the jet is supplied by the accretion disk. But it is reasonable to assume that this matter penetrates in the magnetic field of the central star, partially falls down on the surface of the star and partially is ejected outwards, (Ferreira & Pelletier 1995; Shu et al. 1991). In this case only the magnetic field of the jet is connected with the central star. Schematically this picture of the outflow is presented in Fig. 6. According to this scheme the disk not only supplies the plasma of the jet, but also it produces the magnetized wind which collimates the outflow from the central source into a jet. In Fig. 7 is shown the asymptotic state wherein an inner radially expanding wind is forced to collimate by the surrounding disk wind. A shock wave is formed at the interface of the two components of the outflow, as it may be seen in Fig. (7), (Bogovalov & Tsinganos 2005).

7. Summary

The purpose of this review lecture was to outline the basic theory for studying collimated astrophysical MHD outflows. We highlighted some simple physical results of the theory of MHD outflows. For example the method of nonlinear separation of the variables of the governing MHD equations via the self-similarity assumption has been shown to unify all existing analytical solutions for astrophysical outflows, such as the Parker purely thermally driven wind, the thermally or magnetically driven stellar winds which can be thermally or magnetically confined, and finally the purely magnetically driven and magnetically confined disk-winds (Vlahakis & Tsinganos 1998). Numerical simulations have shown to reproduce results of the analytical solutions for magnetic self-collimation and a generalization of the fast/slow magnetic rotators theory of the Weber & Davis 1-D modeling to the efficient/inefficient magnetic rotators theory via the existence of an energetic criterion for collimation (Sauty & Tsinganos 1994). With this parameter, one may understand the observed appearance of astrophysical outflows in winds and jets. Finally, we have concluded that the various classes of observed YSO outflows may be understood as a combination of a stellar wind and a disk-wind. Analytical theory provides no room for any other possibility. The precise contribution of each of these two components depends on the stage of evolution of the YSO, with disk-winds dominating in the early phases of star formation and the stellar component left alone in the last ZAMS stage of the star, in the form of the familiar solar/stellar wind, when the disk and its wind have both dispersed. More classes of exact solutions can be examined to test this general theory, together with numerical simulations of MHD outflows, a task that still remains to be pursued.

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