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SSC time-dependent modeling of blazars emission

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Abstract. A time-dependent synchrotron self-Compton (SSC) leptonic model, developed with some analytical formulations and a numerical code is introduced. This SSC emission model allows to investigate quantitatively the multi-wavelength and broad-band spectral variability of blazars, providing estimates for physical parameters and the bolometric power, through fits of the spectral energy distribution.

Key words. quasars: general – BL Lacertae objects: general – methods: numerical – methods: analytical – radiation mechanisms: non-thermal

1. Introduction

The rapid and large-amplitude variability of blazars implies non-stationary emitting particle distributions, and requires a time-dependent modeling. The energetic and flaring blazar emission can be modeled in first approximation, as a one-zone active blob (of size R) into the jet, ignited by a shock, and described therefore by a synchrotron self-Compton (SSC) scenario. The evolution of the relativistic electrons injected with a rate $Q(E)$ [cm^{-3} s^{-1}]) and the time-dependent synchrotron and inverse Compton (SSC) emissivities are described by the model.

2. Kinetic equation and a solution

Stochastic particle transport and statistic motion equation of the particle distribution can be approximated by a diffusion equation in the case of small fluctuations. The truncation at the second order of the integral operators expansion is the Fokker-Planck (diffusion-advection) equation (Chandrasekhar 1943; Risken 1996; Gardiner 2004). It can be used in the spherical, ultra-relativistic case, and for isotropic and homogeneous distributions, to describe the electron distribution $N = N(t, E)$ in the blazar's blob:

$$\frac{\partial N}{\partial t} = \frac{1}{A(E)}\frac{\partial}{\partial E}\left(B(t,E)N + D(t,E)\frac{\partial N}{\partial E}\right) + \\ - \frac{N}{\tau(t,E)} + Q(t,E), \quad (1)$$

Neglecting the escape and loss terms the previous equation assumes the form of a continuity equation: $\partial N/\partial t = \partial F/\partial E$, where $F(t, E) = D(t, E)(\partial N/\partial E) + B(t, E)N$ is the 1D particle flux. The no–flux boundary condition is appropriate in our modeling (particles cannot gain energy without limits, because sources are finite and cooling mechanisms

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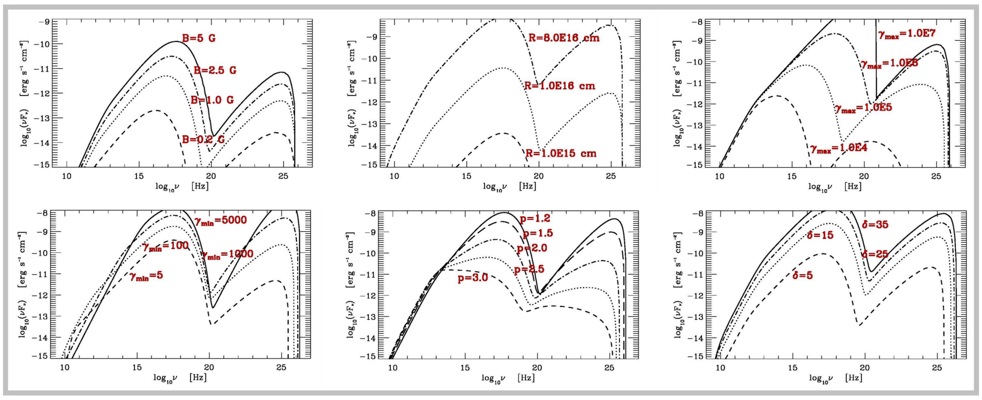

Fig. 1. Synchrotron self-Compton (SSC) spectral energy distributions (SEDs) produced by the numerical code for different changes of parameters (magnetic field intensity B, region size R, maximum Lorentz factor of the injected electron distribution γ_{max}, minimum Lorentz factor of the injected electron distribution γ_{min}, energy index of the power law electron distribution injected p, bulk Doppler factor of beaming $\mathcal{D}$) (From Ciprini (2008)).

win at high energies). In the "hard-sphere" plasma turbulence approximation (accelerated electrons scatter off field baring turbulent elements idealized as randomly moving hard-spherical scattering centers) gives: coefficient $D(t, E) \propto E^2$, coefficient $B(t, E) \propto E^2$ and the characteristic loss rate time $\tau(t, E) = cost$ (Parker & Tidman 1958; Drury 1983), and some time–dependent solutions were found already by Kardashev (1962). Considering an instantaneous gain of energetic electrons with synchrotron and Inverse Compton (IC) cooling, escape with no diffusive processes $D(t, E) = 0$, and expressing all in terms of the non-dimensional Lorentz factor $\gamma = E/m_e c^2$, the equation for $N(t, \gamma)$ can be reduced to:

$$\frac{\partial N}{\partial t} = b\frac{\partial}{\partial \gamma}\left(\gamma^2 N\right) + Q(t, \gamma) - \frac{N}{\tau(\gamma)}, \qquad (2)$$

where $-\dot{\gamma} = b\gamma^2 = \frac{4}{3}\frac{\sigma_T c}{m_e c^2}\left(U_B + U_{rad}(t, \gamma)\right)\gamma^2$ [s^{-1}], denoting with $\dot{\gamma}$ the total synchrotron and IC (SSC), cooling rate, and being $U_B = B^2/(8\pi)$ (B magnetic field strength here) and $U_{rad}(t, \gamma)$ (the photon energy density). The cooling timescale is defined by: $t_{cool} = -\gamma/\dot{\gamma} = (m_e c^2)/\left(4/3\sigma_T c\gamma(U_B + U_{rad})\right)$

If the injection is time independent then $Q = Q(\gamma)$ and the electrons cool completely before to escape ($t_{esc} \gg t_{cool}$), the distribution N achieves the equilibrium and results: $N(\gamma) = -1/\dot{\gamma} \int_{\gamma}^{\gamma_{max}} d\gamma' \, Q(\gamma')$.

Some analytical solutions to time dependent equations can be found (for example Park & Petrosian 1995), even if a more complete treatment is possible only with numerical codes. The equation (2) can be solved with the method of characteristics and the Green function formalism, in the case of an isotropic pith angle distribution and neglecting the escape term ($t_{esc} \to \infty$): $\partial N(t, \gamma)/\partial t - \frac{\partial}{\partial \gamma}(B(t, \gamma)N(t, \gamma)) = Q(t, \gamma), \quad \Rightarrow \quad \mathcal{L}N(t, \gamma) = Q(t, \gamma)$, with initial value $N(0, \gamma) = f(\gamma)$. The kernel of the equivalent integral equation (Green function), provides a formal solution of our partial differential equation (in terms of the general integral $N_0(t, \gamma)$ of the homogeneous associated equation $\mathcal{L}N_0 = 0$, and a particular solution of the complete, inhomogeneous equation). Solutions to the initial value homogeneous problem (i.e. a mere 1D continuity equation) can be obtained with the methods of the characteristics of Lagrange, reducing it to a system of ordinary differential equations. The characteristic parametric equivalent system, we obtain the solution $N_0 = f(\gamma - (dF/dN_0)t)$. In our case $B(t, \gamma) = -\dot{\gamma} = b\gamma^2$, and choosing the initial condition

$N_0(0,\gamma) = f(\gamma) = k\gamma^{-(\alpha+1)}$, we can write the homogeneous solution:

$$N_0(t,\gamma) = k\gamma^{-(\alpha+1)}(1+b\gamma t)^{-(\alpha+1)}. \quad (3)$$

A particular solution of the complete non-homogeneous equation $\mathcal{L}N = Q(t,\gamma)$ can be found using the equivalent equation satisfied by the Green function for the differential operator $\mathcal{L}$ (i.e. the 1D continuity equation): $\mathcal{L}G = \mathbb{I} \Rightarrow \frac{\partial G(t,t',\gamma,\gamma')}{\partial t} - \frac{\partial}{\partial\gamma}(B(t,\gamma)G(t,t',\gamma,\gamma')) = \delta(t-t')\delta(\gamma-\gamma')$, with $B(\gamma) = -\dot{\gamma} = -d\gamma/dt$, and $t = -\int 1/B(\varepsilon)d\varepsilon = t(\gamma)$. Resolving the integral, inverting the relation, and using the derived characteristic curves $\gamma(t) = \gamma'/(1-b\gamma' t)$, and multiplying the the equation by coefficient $B(\gamma)$, an ordinary differential equation is obtained. This is solved with a path integral along the characteristic gamma curve, parameterized with the curvilinear coordinate $\gamma = \gamma(t)$ obtaining: $G(t,t',\gamma,\gamma') = \frac{Q_0\gamma'^{-(\alpha+1)}}{b(\alpha-1)}\left(1-(1-b\gamma t)^{\alpha-1}\right)$. The total solution of equation (2) is:

$$\begin{aligned} N(t,\gamma) &= k\gamma^{-(\alpha+1)}(1+b\gamma t)^{-(\alpha+1)} + \\ &+\frac{Q_0\gamma^{-(\alpha+1)}}{b(\alpha-1)}\left(1-(1-b\gamma t)^{\alpha-1}\right). \end{aligned} \quad (4)$$

When $k = 0$ ($N(0,\gamma) = 0$) we have

$$\begin{aligned} N(t,\gamma) &= \frac{Q_0\gamma^{-(\alpha+1)}}{b(\alpha-1)}\left(1-(1-b\gamma t)^{\alpha-1}\right) \\ &\simeq \begin{cases} Q_0\gamma^{-\alpha}t, & \gamma \ll \frac{1}{bt} \\ \frac{Q_0\gamma^{-(\alpha+1)}}{b(\alpha-1)}, & \gamma \gg \frac{1}{bt} \end{cases}, \end{aligned} \quad (5)$$

where $b = B_{syn+com}/\gamma^2 = (4/3)(\sigma_T c/m_e c^2)\,(U_B + U_{rad}(t,\gamma))$, corresponding to the solution found by Kardashev (1962) for coupled processes of cooling and injection.

3. Synchrotron and SSC spectra

The synchrotron emissivity of a single particle $j(\omega,E,\alpha)$ [erg s^{-1} sr^{-1} Hz^{-1}], is given by (Pacholczyk 1970; Rybicki & Lightman 1979): $j(\omega,E,\alpha) = \frac{\sqrt{3}e^3B}{2\pi m_e c^2}\sin\alpha\ \frac{\omega}{\omega_c}\int_{\frac{\omega}{\omega_c}}^{\infty} d\eta\ K_{\frac{5}{3}}(\eta) = \frac{\sqrt{3}e^3B}{2\pi m_e c^2}\sin\alpha F\left(\frac{\omega}{\omega_c}\right)$, where α is the pitch angle, $K_u(\eta)$ are the hyperbolic Bessel functions of order u, defined with pure imaginary argument, where $F(x) = x\int_x^{\infty} d\xi K_{\frac{5}{3}}(\xi)$, and where $\omega_c = \frac{3\gamma^2 eB}{2mc}\sin\alpha = \frac{3}{2}\gamma^2\omega_L\sin\ \alpha = \frac{3}{2}\gamma^3\omega_B\sin\ \alpha = 2\pi/T_c$ is the critical angular frequency. Supposing a completely random (tangled) magnetic field (homogeneous, isotropic), we can neglect polarization and use an isotropic emissivity $J_{iso}(t,\omega)$, as well as the average on the pitch angle values α and on the azimuth angle φ values, of the total emissivity $J_{hom}(\omega,\alpha)$, summed on all the polarizations in an homogeneous and isotropic magnetic field $\mathbf{B}$: $J_{iso}(t,\omega) = \frac{1}{4\pi}\int_0^{2\pi} d\varphi\int_0^{\pi} d\alpha\ \sin\alpha\left(\frac{1}{4\pi}\int_0^{\infty} dE\ N\ j_{iso}(\omega,E,\alpha)\right) = \frac{\sqrt{3}e^3B}{2\pi m_e c^2}\frac{1}{4\pi}\int_0^{\infty} dE \qquad N\left(\frac{1}{2}\int_0^{\pi} d\alpha\ \sin^2\alpha\ F\left(\frac{2\varpi}{sin\alpha}\right)\right)$, where $N = N(t,E)$, $\omega_L = (eB)/(mc)$ is the Landau angular frequency, $\omega_B = \omega_L/\gamma$ the cyclotron angular frequency for Landau quantum level, and $\varpi = \omega/(3\omega_L\gamma^2)$ the normalized angular frequency. This integral can be handled integrating by parts in $d\alpha$, using the substitution of variables $\cos\alpha = \sinh\beta/\cosh\beta$, then solving in the complex domain the integrals containing the K_u functions and their algebraic properties. The resulting single electron emissivity averaged over pitch angles α, to be convolved with the electron energy distribution N in the isotropic case is:

$$\begin{aligned} j_{iso} &= \frac{3\sqrt{3}\sigma_T c U_B}{\pi\omega_L}\varpi^2\cdot \\ &\cdot\left(K_{\frac{4}{3}}(\varpi)K_{\frac{1}{3}}(\varpi) - \frac{3}{5}\varpi\left(K^2_{\frac{4}{3}}(\varpi) - K^2_{\frac{1}{3}}(\varpi)\right)\right), \end{aligned}$$

measured in [erg s^{-1} sr^{-1} Hz^{-1}].

In our SSC modelling the power law injection Q is defined in a finite $(\gamma_{min},\gamma_{max})$ range, thus: $J_{syn}(t,\omega) = 1/(4\pi)\int_{\gamma_{min}}^{\gamma_{max}} d\gamma\ \ N(t,\gamma)j_{iso}(\omega,\gamma)$, and the time dependent absorption coefficient is: $k(t,\omega) = \pi/(m_e\omega)\int_1^{\infty} d\gamma\ \ \frac{N(t,\gamma)}{\gamma p}\frac{\partial}{\partial\gamma}(\gamma p j_{iso}(\omega,\gamma))$ [cm^{-1}], with $p = m_e c\sqrt{\gamma^2-1}$. For a region of dimension R along the line of sight, the optical depth is $\tau(t,\omega) = k(t,\omega)R$, and the self-absorption frequency in which $\tau(\nu_a) = 1$ can be derived, while the intensity is the solution of the radiative transfer equation: $I_{syn}(t,\omega) = \left(J_{syn}(t,\omega)/k(t,\omega)\right)\left(1 - e^{-k(t,\omega)R}\right)$, [erg s^{-1}cm^{-2}sr^{-1}Hz^{-1}].

The IC spectrum is calculated as pure SSC (interaction of electrons with the isotropic synchrotron photon distribution) integrating the single electron scattering spectrum over the electron distribution $N(t,\gamma)$ in a homogeneous region Blumenthal & Gould (1970): $J_{com}^{(SSC)}(t,\nu_f) = \int_{\nu_s^{min}}^{\nu_s^{max}} d\nu_s \int_{\gamma_1}^{\gamma_2} d\gamma N(t,\gamma) j_{com}(\gamma,\nu_s,\nu_f) I_{syn}(\nu_s)$, [erg s^{-1}cm^{-3}sr^{-1}Hz^{-1}], where: ν_s is the frequency of the incident synchrotron radiation; ν_f is the frequency of the outgoing radiation, after the IC scattering; $N(\gamma)$ is the electron distribution in the emitting region; $j_{com}(\gamma,\nu_s,\nu_f)$ is the IC emissivity of the single electron,; $I_{syn}(\nu_s)$ is the intensity of the incident synchrotron radiation; and $\gamma_1 = \max\left[\sqrt{\frac{\nu_f}{4\nu_s}}, \gamma_{min}\right]$, $\gamma_2 = \min\left[\gamma_{max}, \frac{3m_ec^2}{4h\nu_s}\right]$. The IC emissivity of a single electron there, (scattering the monochromatic radiation of frequency ν_s), is defined as Blumenthal & Gould (1970); Rybicki & Lightman (1979): $(N(\gamma)\sigma_T F_0)/(4\gamma^2\beta^2\epsilon_s) f(\epsilon_s,\epsilon_f)$, with $f(\epsilon_s,\epsilon_f) = (1+\beta)\frac{\epsilon_f}{\epsilon_s} - (1-\beta)$ when $\frac{1-\beta}{1+\beta} \leq \frac{\epsilon_f}{\epsilon_s} \leq 1$; with $f(\epsilon_s,\epsilon_f) = (1+\beta) - \frac{\epsilon_f}{\epsilon_s}(1-\beta)$ when $1 \leq \frac{\epsilon_f}{\epsilon_s} \leq \frac{1+\beta}{1-\beta}$ and with $f(\epsilon_s,\epsilon_f) = 0$ elsewhere (being $\epsilon = h\nu$ and $\beta = v/c$). When the homogeneous region of dimension R is fully transparent to the IC frequencies, we can set $I_{com}(t,\nu_f) = J_{com}(t,\nu_f)R$.

The total spectrum of the synchrotron and the inverse Comtpon radiation is: $I_{tot}(t,\nu) = I_{syn}(t,\nu) + I_{com}(t,\nu)$ and the luminosity (bolometric power) $L(t,\nu) = (16\pi^2/3)R^2 I_{tot}(t,\nu)$. The observed quantities corrected for the relativistic beaming and cosmological effects are: $\nu_{obs} = \mathcal{D}\nu$, $L_{obs}(t,\nu_{obs}) = (\mathcal{D}/(1+z))^3 L(t,\nu)$, where $\mathcal{D} = 1/(\Gamma(1-\beta\cos\theta))$ is the bulk Doppler beaming factor.

4. Numerical implementation

The algorithm for the solution of the time-dependent kinetic equation consists of a direct method resolving the equivalent system of algebraic discretized equations obtained with a one step finite difference scheme. A fast and robust semi-implicit conservative scheme of Chang & Cooper (1970). This numerical method is the best compromise between functionality, stability and accuracy. Intervals Δt and $\Delta\gamma$ of the two-dimensional mesh representing the electron number density function $N(t,\gamma)$, can be rather broad, and the semi-implicit method provides accurate time-dependent solutions (and non-negativity of solutions). The energy (γ factor) has a logarithmic equally spaced mesh, while the time a linear mesh. After the solving of the equation, at every discrete time step the synchrotron emissivity and intensity, the IC (SSC) intensity, and the total intensity and luminosity are calculated based on the analytical formulations summarized in the previous sections. Beaming effects are then evaluated as well (Ciprini 2008).

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