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Statistical techniques for interferometric signal analysis

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Abstract. Phase disturbance due to atmospheric turbulence affects fringe tracking. Algorithms aimed at fringe parameter identification are based on interferometric models that have to be carefully adapted to the interfered beams, including sources of variability. All the information is contained in the collected signals, and how to extract it is a major research problem.
This work aims to determine some stochastic properties of the signals from the data that are useful for modelling. We apply statistical techniques to real interferometric signals. We examine the composition of signals before and after the combination using time series analysis, and we separate pure random effects and peculiar features from trends. Multivariate regression analysis allows us to isolate noise components due to the interference physical process.

Key words. interferometry; fringe tracking; spectral signal analysis; statistical tools

1. Introduction

Optical and infrared nterferometry is increasingly employed in the study of cosmic objects. The technological aspects have significantly developed, but the growing experience requires continuous refinements. In particular, to observe sources at limiting magnitudes requires fringe tracking. Its role is to check the residual differential path (OPD) between the interfering beams and to correct it, to optimize the sensitivity of the instrument and to expand the integration times (see, e.g., Michelson School 2000).

In the framework of ESO VLTI (Very Large Telescopes Interferometer), the Astronomical Observatory of Turin was involved in the design and construction of a fringe tracker, FINITO (Fringe-tracker Instrument of NIce and TOrino), and a fringe sensor for the PRIMA instrument (Phase Referenced Imaging and Micro-arcsecond Astrometry) (Gai et al. 2004). One of their tasks is the evaluation of the OPD in real time, thanks to algorithms able to recover fringes information from raw data. All algorithms are based on a model of the interferometric signal, so the algorithms' performance are dependent from the accuracy of the model knowledge.

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The practise has shown that theoretical models should be tailored to fit real data. Different sources of noise add to signals, such as atmospherical effects in terms of refractive indeces and especially turbulence, plus instrumental or environmental noises. Moreover, instrumental characteristics, such as transmission, phase, quantum efficiency, are wavelength–dependent and have to be described using spectra distributions. Their fluctuations influence the system stability and should be measured with calibration systems. The aim of the present work is to analyze possible causes of discrepancies between measured and reconstructed signals using statistical techniques. In Sec. 3 we describe the involved signals recognizing their different components and their individual and mutual properties. Sec. 4 discusses how to understand the output variability in terms of input variability. For a more extensive discussion of these topics see (Bonino 2009).

2. Data description

The analysis has been performed using sets of measurements acquired by the VLTI VINCI (VLT INterferometer Commissioning Instrument) instrument (Kervella et al. 2000), working in K band ([2.0 – 2.5] μm). This provides:

- synchronous photometric and interferometric recording;
- real raw data, without OPD correction;
- availability of a significant amount of data.

Each observation produces four data sets. The first three are for calibration: the stellar flux can be propagated into a single channel, while the other is left open, or both arms of the interferometer can be left open. These operational modes do not produce fringes in the outputs, but can be used for characterization of the interferometer noise with no or single input. Finally, the fourth set contains the results of the interference of both beams.

Each set consists of several OPD scans, each composed of four time series, two for the photometry and two for the interferometry. The flux intensities are given in ADU (Analog Digital Unit) and they are integers. An example of interferometric record is given in Fig. 1. The evident photometric fluctuation is also reflected in the interferometric outputs.

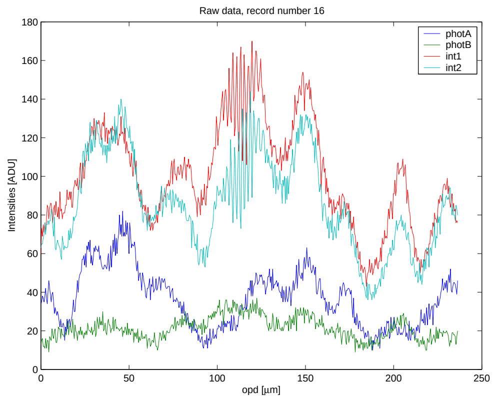

Fig. 1. Raw data, record 16 of case 4. From bottom to top: photometric channel P_A, phot. channel P_B, interferometric channel I_2, interf. channel I_1. It is visible the central modulated part of interferometric channels where fringes effectively form.

3. Statistical signal analysis

Techniques of classical time series analysis (auto/cross–correlation coefficients, power spectral density function) supply information on the structures of interferometric signals. A connection between time and frequency features is given by the Allan variance (Allan 1966).

For each data set, we evaluate the autocorrelation and cross–correlation coefficients for each record using their sampling estimators, the auto/cross–correlogram (see, e.g., Priestley 1981, page 321). The coefficients are then averaged over all records.

The analysis of calibration data highlights the presence of two different components on the signals: a linear 'trend', that contains large amplitude fluctuations, which we call 'macroscopic', and a residual variability, that we define 'microscopic'. The former is responsible for the presence of a slowly decreasing autocorrelation (and cross–correlation between different channels). In fact, if we subtract the trend from raw data, all auto/cross-correlations decay immediately to zero. The microscopic component is an uncorrelated process. Similarly, interferometric signals present a non–negligible autocorrelation at low lags, due to photometric fluctuations; if we subtract them from signals, we get an harmonic process, i.e., 'clean' fringes.

The presence of macroscopic fluctuations can cover specific properties of the signal. Indeed the autocorrelation coefficients before the subtraction of fluctuations are similar for both photometric and interferometric signals.

Cross–correlation between photometric and interferometric signals, i.e. beams before and after the combination, suggests that the interference process does not introduce other anomalous features on interferometric outputs.

These results are confirmed by the power spectral analysis, since the power spectral densities (PSDs) are affected by low-frequency components that are linked to the slow drift in the data. Once this trend is subtracted, the PSDs confirm that the residuals are uncorrelated signals, aside from frequencies around the zero that may indicate some leakage of the zero-frequency component.

Photometric fluctuations are quite difficult to interpret. A characterization of the photometry temporal behaviour is given by the Allan variance, which considers the variance of samples separated by a certain time lag. This technique shows that photometric signals can be considered locally uncorrelated over ≈ 10-samples-sized intervals. We use this information for the proper subtraction of fluctuations. At the working wavelength of VINCI ($\sim 2.0\ \mu m$), the photometric fluctuations are stable over 6.9 msec, or equivalently 4.5 μm, i.e. two fringes.

4. The interference process and its variability

We have seen in the previous Section that some information can be retrieved from the signals themselves, with statistical moments or correlations. However, the availability of data from photometric channels, i.e. the interferometric system inputs, allows us to investigate if the input variability is sufficient to explain that at the output, or if we should suspect another variability source, perhaps an instrumental contribution.

Let us introduce a simple model for the analyzed signals. For photometric channels P_A and P_B, we separate the 'true' unknown values and their variations $\epsilon_\bullet(x)$:

$$\begin{aligned} P_A(x) &= \tilde{P}_A(x) + \epsilon_{P_A}(x) \\ P_B(x) &= \tilde{P}_B(x) + \epsilon_{P_B}(x). \end{aligned} \quad (1)$$

Here x is the spatial variable for the optical path difference. When PA and PB are physically combined to produce the interferometric channels I_1 and I_2, also the noises $\epsilon_{PA}(x)$ and $\epsilon_{PB}(x)$ contribute to the variability on the interferometric outputs, say $\epsilon_{I_1}(x)$ and $\epsilon_{I_2}(x)$:

$$\begin{aligned} I_j(x) &= [\beta_{j,A} PA(x) + \beta_{j,B} PB(x)] \cdot \\ &\cdot\ [1 + m_j(x)] + \epsilon_{I_j}(x), \quad j = 1, 2 \end{aligned} \quad (2)$$

where $m_\bullet(x)$ are the modulation functions of the fringes, and $\beta_{\bullet,\bullet}$ are the coefficients of the photometric inputs. We wonder if the introduction of ϵ_{PA} and ϵ_{PB} in the combination process is sufficient to explicate ϵ_{I1} and ϵ_{I2}. The ideal combination is noiseless. However, we expect some kind of superposed noise, caused by the physical instruments used for the composition/separation of beams (fibres in this case, or optical combiners). We assume here that the noise due to the combination process is uniformly present on the data, and we wish to quantify its weight.

We use the multivariate regression analysis (see, e.g., Rawlings et al. 2001). This choice is suggested by the interferometric model itself (eq. 2), that also addresses the use of a linear model, when one disregards the modulation component. We examine two different regression models, a linear one and a 'mixed' linear one, i.e. with a higher order factor ($PA \cdot PB$) to describe the non-linearity of the interferometric combination. Let us describe the main steps of our analysis:

1. we select records with homogeneous variance over all channels. For each record, we eliminate the coherence length area, where the modulated part $m(x)$ dominates the photometric offset;

2. for both models, we perform tests on the coefficients of the regression model to see if they were null, on the residual unexplained variance, and we study the residuals, i.e. the differences between real values and those predicted by the model.

The analysis has been carried out with the software STATISTICA by StatSoft.

The analysis of the multiple correlation coefficients of both models reveals that the variability on the inputs explains almost all the variability at the outputs. The unexplained variability results very small. Physically, we can associate this with the instrumental contribution in the interference process to the total noise of the system. Its low magnitude suggests that the system does not add important perturbations to the outputs.

This unexplained variability accounts for $< 1\%$ of the total. This result is substantially the same for both regression models, the linear and the 'mixed' one. The difference between the two models lies in the characteristics of their residuals. We can be confident that for both models the residuals are normally distributed random variables, with tails that are smaller for the 'mixed' model. Furthermore, the variance of the residuals of the linear model has some inhomogeneity that is not present in the variance of the 'mixed' model residuals. This fact suggests that the 'mixed' model is preferred.

The higher order term $PA \cdot PB$ has a physical interpretation as the residual of the modulation outside the coherence length, and gives the non–linearity of the interference process. We note a quite unexpected result of our model. The starting point of our study is the selection of data characterized by homogeneous variance. This choice seems to correspond to instances where the noise covers the modulation pattern, i.e. cases where the noise linearizes the model. However, due to the mixed term in the regression analysis, we recover this non–linear part of the signal, separating it from the linear part and from the global noise. This ability could be very useful in low SNR context, i.e. when fringes are comparable or even smaller than noise.

5. Conclusions

The analysis performed using real interferometric signals shows the importance of photometric fluctuations that are difficult to model. They can be handled off-line, but they could be a problem when working in real time, such as in fringe tracking mode. We have shown in Sec. 3 that they can mask all peculiar features of signals, and they can worsen the performance of OPD location algorithms. Use of statistical tools, such as Allan variance, which links temporal and spectral features, or the multivariate regression, allows to face these difficulties. In particular, the latter can be used in demanding situations, such as in low SNR region. These tools would be very helpful, either on–line or off–line, both for the diagnostics of operating conditions of instruments and for data calibration. They can be easily adapted to different interferometric instruments.

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