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Mutual interactions of two planetary objects in a protoplanetary accretion disc: 2D SPH simulations

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Abstract. We present here a study based on the migration of protoplanets in an accretion disc of a forming star, as the mainly proposed scenario for the formation of planetary systems. Attention is here focused on the mutual interactions between two protoplanets, both embedded in the accretion disc, as a function of the protoplanets masses, their relative positions, the dynamic properties of the accretion disc particles. The study is performed through a 2D SPH code and preliminary results show an oscillation of the distance between the two protoplanets, together with a slow migration of the two planets towards the central star when two Jupiter-like planets are considered. Less correlated behaviour is observed when at least one of the two protoplanets has an Earth-like mass. The role played by the disc particles initial angular momentum is discussed.

1. Introduction

Currently, nearly 300 extra-solar planets have been detected (http://www.exoplanets.eu/), and most of them are massive (Jupiter-like) planets with small (fractions of AU) semi-major axes (Perryman, 2000; Udry & Santos, 2007). Theoretical models for the formation of planetary systems are mostly based on the crucial role of an accretion disc around a forming young star, where the disc provides both the material for the forming planets and the interactions responsible for the migration of the protoplanets towards the central star. The need for protoplanetary migration comes from the difficulty of an "in situ" formation in such close to the star positions, so that the detailed understanding of migration mechanisms appears mandatory in order to build self-consistent models for planetary systems formation. Some analytic studies on the matter (Ward (1997) and references therein) suggest that gravitational torques due to resonant non-axisymmetric structures could act to transfer angular momentum between the protoplanets and the disc. This disc-planet interaction induces also a repulsion of material on either side of the protoplanets orbits, with a possible formation of a density gap, depending on the interaction strength. Two main migration types are then distinguished: *type I*, for Earth-like planets, and *type II* for "massive" (Jupiter-like) protoplanets (Ward, 1997).

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In both cases, gravitational torques and viscosity play a major role in determining the protoplanets migration rate, and it is possible that the migration lifetime is shorter than the lifetimes of planet formation or disc dissolution, causing the collapse of the planet toward the star. Several papers devoted to this theme, according to various fluid-dynamics schemes (Artymowicz, 2004; D'Angelo et al., 2003, 2006; Kley, 2000; Papaloizou et al., 2007; Schaäfer et al., 2004), showed that this mechanism should be very effective for the protoplanet dragging toward the central protostar in characteristic time-scales of the order of 10^5 years.

Most models assume isothermal accretion disc, with an initially Keplerian gas velocity distribution. Here, following a preliminary study on the effect of sub-Keplerian conditions on protoplanetary, inviscid disc evolution (Costa et al., 2010), we study the role of planet-disc dynamic interactions and of mutual planet-planet interaction on the protoplanet orbital evolution, during the initial formation phases of stellar systems.

In this work we show some preliminary results obtained with several sub-Keplerian disc models, focusing our attention on the effects due to planet-planet interaction in planetary migration both for Jovian and for Terrestrial protoplanets. Models of Jovian or Terrestrial planets, interacting with an initially Keplerian accretion disc, are also produced for comparison.

2. Models features and boundary conditions

The simulations presented here are developed with a 2D Cartesian model based on the Smoothed Particle Hydrodynamics (SPH) numerical method (Monaghan, 1992). The choice of the SPH method is funded on the grounds of its Lagrangian nature and its ability to easily tackle hydrodynamic problems with free surfaces. The code uses the following equation of state: $P = (\gamma - 1)\rho u$, where u is the thermal energy per unit mass, and the polytropic index γ can be adjusted to values lower than 5/3 if radiation, partial molecular dissociation or partial ionisation effects are present, giving the overall effect of a higher compressibility to the gas. Current calculations are performed with the simple choice of $\gamma = 5/3$.

The model is built with dimensionless quantities. Reference physical units are (Costa et al. (2010) for details): the initial stellar mass M_0; the stellar radius R_0; the Keplerian period of an orbit of radius R_0 around a star of mass M_0, $T_0 = \frac{2\pi}{\sqrt{GM_0}} R_0^{3/2}$.

The initial conditions used to build the accretion disc are axially symmetric. The accretion disc is in fact created through the injection of particles at point-like positions (injectors) along a circle concentric with the central star. Particles have initial tangential velocities set by choosing a value for the angular momentum per unit mass j. Three values were chosen for the specific angular momentum j: 18, 36, 54, for particles injected at a radial distance of 130. They all give a sub-Keplerian feature to the velocity distribution of the accretion disc (particles injected at a radial distance of 130 would have Keplerian velocity with $j \sim 70$), so that injected particles start decreasing their distance to the centre, until they "hit" the centrifugal barrier, possibly generating shock waves. At some point, the accretion disc reaches a nearly steady state population of particles (time-averaged, since the turbulent nature of the gas flow does not allow an actual steady state) with $3 \times 10^5 - 5 \times 10^5$ particles, depending on the model, when using a smoothing length (spatial resolution parameter of the SPH method) $h = 0.3$. At this stage, two planets are inserted and the model is evolved considering the following interactions: gravitational interaction between gas particles and central star; gravitational interaction between planets and star; gravitational interaction between the two planets; gravitational interaction between particles and planets; gas pressure between neighbouring gas particles; artificial viscosity (pressure contribution) between gas particles; gas particle capture by the planet (conservation of momentum is used to correct the planet speed before removing the particle) when gas particles approach the planet area (planet-particle distance less than $2h$).

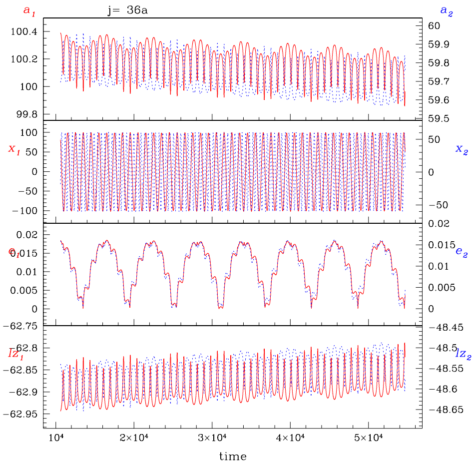

Fig. 1. Orbital parameters evolution for model 36a. The semi-major axis a, the x coordinate of the protoplanets, the eccentricity e and the specific angular momentum l_z are reported, as a function of time, on the top, mediums and bottom rectangles respectively. The blue (dotted line) and red (solid line) graphs are for the two protoplanets included in the simulation.

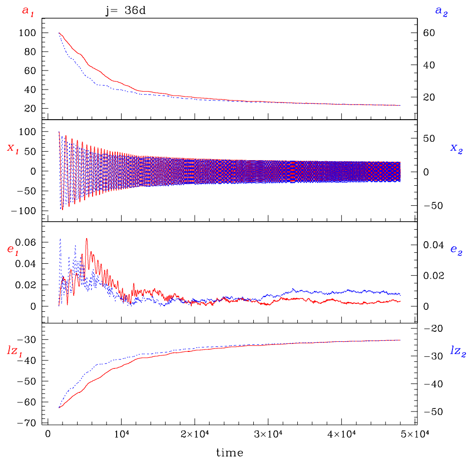

Fig. 2. Orbital parameters evolution for model 36d. See fig. 1 caption for description.

No self-gravitation is included in the gas for computational reasons, and this is not believed to negatively affect our computations since the disc total mass is $10^{-11}\times$ the number of particles, which for $\sim 10^5$ particles means 10^{-6} (in units of the central star mass M_0), so that the dominating gravitational force is still that of the central star, and that generated by the two protoplanets as a second order effect. The models are inviscid (artificial viscosity only), because we are here focused on the protoplanet migration mechanism based on gravitational and pressure forces and momentum transfer through gas particle captures, rather than to viscosity driven migration. Moreover, mutual interaction between the two inserted planets is our main topic here.

Calculations are performed with two planet masses: 10^{-7} (Earth-like) and 10^{-3} (Jupiter-like).

The computational domain is circular, extended in the xy plane with a radius of 150 (the star is located in the "origin"), and the planets are located inside the disc plane, with an initial zero eccentricity. The planets are initially located in the x axis at distances of 100 and 70 respectively.

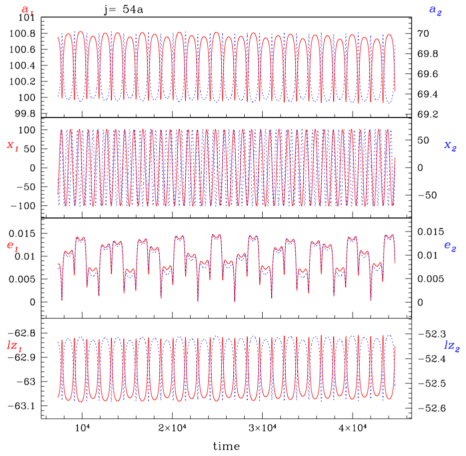

Fig. 3. Orbital parameters evolution for model 54a. See fig. 1 caption for description.

In order to compare some of our results with more traditional models (Ward, 1997), some computations are built using an initial exactly Keplerian distribution of gas particle velocities, with an initial uniform density over the entire disc area.

Table 1. List of computed models, with their identifying labels (first column). The second column gives the specific angular momentum of injected gas particles, the third and fourth columns give the initial masses of the protoplanets in units of the central star mass.

Label	j_p	M_1	M_2
18a	18.0	10^{-3}	10^{-3}
18b	18.0	10^{-3}	10^{-7}
18c	18.0	10^{-7}	10^{-3}
18d	18.0	10^{-7}	10^{-7}
36a	36.0	10^{-3}	10^{-3}
36b	36.0	10^{-3}	10^{-7}
36c	36.0	10^{-7}	10^{-3}
36d	36.0	10^{-7}	10^{-7}
54a	54.0	10^{-3}	10^{-3}
54b	54.0	10^{-3}	10^{-7}
54c	54.0	10^{-7}	10^{-3}
54d	54.0	10^{-7}	10^{-7}
kepl-a	71.6	10^{-3}	10^{-3}
kepl-b	71.6	10^{-3}	10^{-7}
kepl-c	71.6	10^{-7}	10^{-3}
kepl-d	71.6	10^{-7}	10^{-7}

A summary of the distinguishing features of our models is given in table 1 together with identifying labels.

3. Results

Figures 1 - 3 show the evolution of the orbital parameters for four among the models reported in table 1, the other being still under development and analysis. The coordinate x is plotted to allow us counting the protoplanet orbits. The main feature emerging from the sub-Keplerian model 36*a* and 54*a* is a downward migration of the protoplanets, together with an oscillation of the semi-major axes of the two planets , with strongly correlated periods. We expect the migration speed to be dependent on planets mass. Model 54*a* shows a slower migration, as expected on grounds of the higher j values of disc gas particles (Costa et al., 2010). In fig. 2 the preliminary results for model 36*d*, which includes two terrestrial planets, are shown. As expected we see a much weaker coupling between the two orbits. Results are more similar to those presented by Costa et al. (2010) for a single protoplanet embedded in a sub-Keplerian accretion disc. The oscillation of a observed with the Jovian planets is due to the planet-planet gravitational interaction. But to better understand eventual indirect effect (modification of the disc structure due by one planet and affecting the other), we need a deeper analysis involving a the whole set of data and it will be presented in a forthcoming paper.

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