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# 2D SPH simulations of a single planet migration in a protoplanetary disc

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**Abstract.** Migration of protoplanets inside an accretion disc of a forming star is the most probable scenario for planetary system formation according to current models. Unsolved problems exist, concerning migration times and mechanisms. We report here the results of a 2D hydrodynamic study within an SPH scheme, analysing migration of an Earth-like or a Jupiter-like planet inside an inviscid sub-Keplerian accretion disc, as a function of the initial specific angular momentum of the infalling accretion disc matter. Particle capture by the protoplanet causes a rapid migration, within a few orbits, for the Earth like planet, and about $10^4$ orbits for the Jupiter like planets. The effect of a planet pseudo-atmosphere is also discussed.

## 1. Introduction

Of the nearly 300 extra-solar planets currently detected (http://www.exoplanets.eu/), most are massive (Jupiter-like) planets with small (fractions of AU) semi-major axes (Perryman 2000; Udry & Santos 2007). The formation of giant planets so close to the central star appears unlikely. Then "migration" processes are crucial for a self-consistent model of planetary systems formation. Some analytic studies on the matter (Goldreich & Tremaine 1980; Lin & Papaloizou 1986; Ward 1997) suggest that gravitational torques due to resonant non-axisymmetric structures could act to transfer angular momentum between the protoplanets and the disc, and that density gap near the planet's orbit may form, depending on the interaction strength. Two main migration types are then distinguished: *type I* (Tanaka et al. 2002), for Earth-like planets; *type II* when a massive protoplanet, capable of opening and maintaining a density gap, remains trapped in this gap. In both cases it is possible that the lifetime of migration is shorter than the lifetimes of planet formation or disc dissolution, causing the collapse of the planet toward the star. Recent studies and numerical simulations give time-scales of the order of $10^5$ years (Artymowicz 2004; D'Angelo et al. 2006; Kley 2000; Papaloizou et al. 2007; Schäfer et al. 2004) for migration of proto-

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planets towards the central stars with these mechanisms. Most models present in literature assume Keplerian velocity distribution of gas particles and isothermal equation of state. Instead, the role of sub-Keplerian flows, pressure forces and turbulence is analysed here. The Keplerian constraint is usually sustainable on the grounds of the absence of pressure and other forces. Pressure forces and interaction with the protoplanet can break any initially Keplerian distribution of the gas particle velocity. We then span a range of possible values for the initial specific angular momentum of the gas, from extreme to soft sub-Keplerian conditions, and track the evolution of the orbital parameters of one protoplanet.

## 2. Models

This work is developed with a 2D Cartesian code based on the Smoothed Particle Hydrodynamics (SPH) numerical method (Monaghan 1992). The code uses dimensionless quantities, referred to the following units: the initial stellar mass $M_0$; the stellar radius $R_0$; the Keplerian period of an orbit of radius $R_0$ around a star of mass $M_0$, $T_0 = \frac{2\pi}{\sqrt{GM_0}} R_0^{3/2}$. More details on the code can be found in Costa et al. (2010).

Particles are injected at point-like positions (injectors) along circles concentric with the central star, with initial tangential velocities set by choosing a value for the specific angular momentum $j$. The planet is inserted when the accretion disc reaches a "nearly" steady state population of particles (time averaged since variations of the number of particles are actually observed during our simulations, due to the highly turbulent nature of disc flows) with $3 \times 10^5 - 5 \times 10^5$ particles, depending on the model ("smoothing length" $h = 0.3$). Together with gravitational interactions (no self-gravitation of the gas), gas pressure, artificial viscosity and gas particle capture by the planet (with momentum conservation) are included. Additionally, in some of our models, an SPH particle is given the planet position and velocity. This allows a pressure driven interaction between the protoplanet and the gas. This approach is indicated as adding a "pseudo-atmosphere" to the protoplanet.

Physical viscosity is not considered, since we are here focused on the protoplanet migration mechanism based on gravitational and pressure forces and momentum transfer through gas particle captures. Moreover, the absence of physical viscosity enhances the turbulent nature of our simulation, allowing us to correctly describe some effects due to turbulence like the development of strong shock waves, especially for low values of the specific angular momentum $j$. Three choices are made for $j$: 18, 36, 54, for particles injected at a radial distance of 130. All of them give a sub-Keplerian profile to the velocity distribution of the accretion disc (particles injected at a radial distance of 130 would have Keplerian velocity with $j \sim 70$). Particles start decreasing their distance to the centre, until they "hit" the centrifugal barrier. The mass of the particles is set to $10^{-11}$ (in units of $M_0$), which means that the total mass of the disc can be about $10^{-6} - 10^{-5}$. The chosen masses for the planet are : $10^{-7}$ (Earth-like planet) and $10^{-3}$ (Jupiter-like planet). The computational domain is circular, extended in the $xy$ plane with a radius of 150 (the star is located in the "origin"), and the planet is located inside the disc plane, with the initial position at $X_p = 100$ and $Y_p = 0$, and an initial zero eccentricity.

A summary of the distinguishing features of our models is given in table 1 together with identifying labels.

## 3. Results

Figures 1-4 show the evolution of the orbital parameters for some of our models. Each figure includes the results from two models, with (models with labels "b" and "d") and without (models "a" and "c") the pseudo-atmosphere. A downward migration of the protoplanet is observed, with a speed strongly dependent on the planet mass. Earth-like planets migrate in a few orbits, while for the Jupiter-like planets, an extrapolation of our results (which are limited to 20-30 orbits), suggests that a few thousands orbits are necessary to move the planets towards the central star.

**Table 1.** List of computed models, with their identifying labels (first column). The second column gives the specific angular momentum of injected gas particles, the third column gives the initial mass of the protoplanet and the fourth column indicates the eventual presence of the pseudo-atmosphere.

| Label | $j$ | $M_p$ | pseudo-atmosphere |
|---|---|---|---|
| 18a | 18.0 | $10^{-3}$ | no |
| 18b | 18.0 | $10^{-3}$ | yes |
| 18c | 18.0 | $10^{-7}$ | no |
| 18d | 18.0 | $10^{-7}$ | yes |
| 36a | 36.0 | $10^{-3}$ | no |
| 36b | 36.0 | $10^{-3}$ | yes |
| 36c | 36.0 | $10^{-7}$ | no |
| 36d | 36.0 | $10^{-7}$ | yes |
| 54a | 54.0 | $10^{-3}$ | no |
| 54b | 54.0 | $10^{-3}$ | yes |
| 54c | 54.0 | $10^{-7}$ | no |
| 54d | 54.0 | $10^{-7}$ | yes |

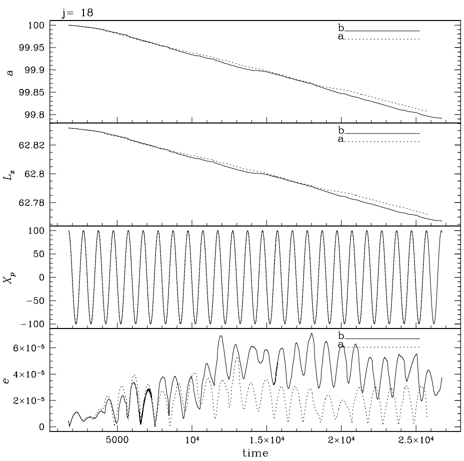


**Fig. 1.** Orbital parameters evolution for models 18a and 18b. The semi-major axis $a$, the specific angular momentum $l_z$ a, the $x$ coordinate of the protoplanet and the eccentricity $e$ are reported, as a function of time. The dotted and continuum graphs are for the two reported data sets, with and without the protoplanet "pseudo-atmosphere".

The above suggested extrapolation requires a linear evolution of the orbital parameters with respect to time. But the results obtained for the less massive planet (see fig. 3) suggest that the migration rate asymptotically decreases.

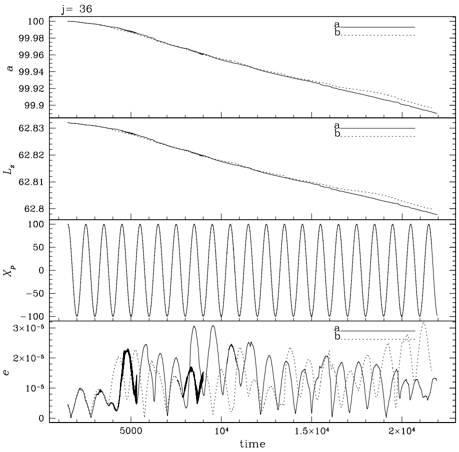


**Fig. 2.** Orbital parameters evolution for models 36a and 36b. See fig. 1 caption for description.

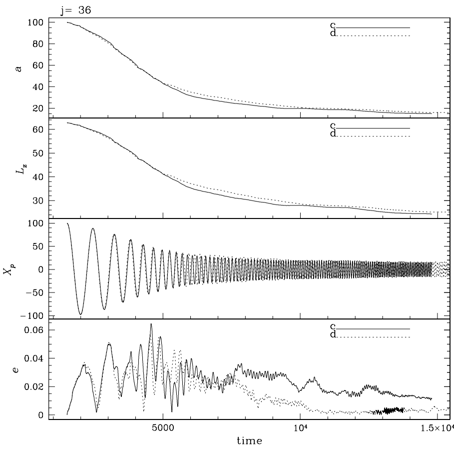


**Fig. 3.** Orbital parameters evolution for models 36c and 36d. See fig. 1 caption for description.

Our disc models show a high value of the Reynolds number ($\sim 300-3000$), evaluated by using the artificial viscosity coefficient $\nu_{num} = c_s h$. This gives our model a highly turbulent behaviour, like the propagation of waves coming from the central zone of the disc, at least for the $j = 18$ and $j = 36$ models. The same is not clearly observed in $j = 54$ models, the cause being probably the high angular momentum of particles. A strong sub-Keplerian condition for

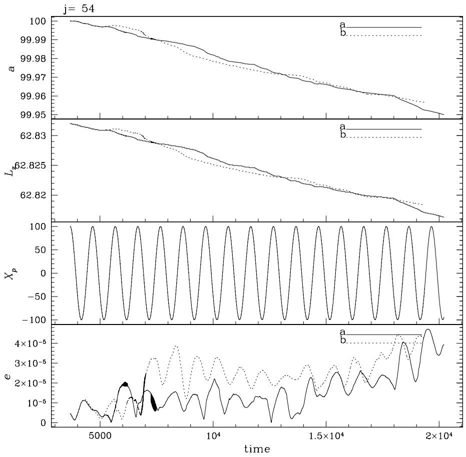


**Fig. 4.** Orbital parameters evolution for models 54a and 54b. See fig. 1 caption for description.

injected particles makes them rapidly fall towards the centrifugal barrier, and the subsequent bouncing-back could be at the origin of these waves (Costa et al. 2010).

In our models the most important contribution to downward migration seems to come from momentum transfer between the gas particles and the protoplanet; the sub-Keplerian $j_z$ value induces an average decrease of $l_z$, until the accretion disc centrifugal barrier is reached. We also believe that the type III migration suggested by Papaloizou et al. (2007) may be present, but it's not the main mechanism. In our case, momentum conservation is directly applied to correct the protoplanet kinematics when disc particles are captured by the planet.

Concerning the pseudo-atmosphere (models *b* and *d*), the difference between the results obtained with and without it is of the order of $\sim 10^{-2}\%$ with the Jovian protoplanet and of $\sim 1-4\%$ with the Terrestrial protoplanet as discussed in Costa et al. (2010). The impact of the pseudo-atmosphere never appears crucial. It adds a repulsive pressure interaction between the protoplanet and the gas, that should hinder gas particle capture by the protoplanet, determining a slightly faster migration rate.

In $c, d$ models the radial migration breaks down close to $\sim 15; 20; 45$ stellar radii, for $j = 18; 36; 54$ respectively. We believe that this effect is mainly due to the interaction of the protoplanet with the propagation of outward gas waves coming from inner zones, where the centrifugal barrier is located (Costa et al. (2010) for details). Of course, due to a much higher inertia, Jovian planets evolve much more slowly.

Concerning the eccentricity, figures 1-4 show that for Jupiter-like planets the orbits remain almost circular, with values around $10^{-5}$, while for Earth-like planets some eccentricity enhancing is visible, but the behaviour is quite complex and a more stable value is obtained in late migration phases, with values in the $0.01-0.02$ range.

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