



Tearing and velocity shear driven instabilities in the heliospheric plasmas: three-dimensional simulations

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Abstract. We have studied magnetic reconnection and shear flow instabilities using a new-compact algorithm recently designed, which combines high-order techniques both in space and time, to follow magnetic reconnection developing in current-sheets, together with shock-capturing capabilities, able to handle field discontinuities often developing in a compressible plasma. In particular we have followed the three-dimensional non-linear evolution of the tearing instability and its transition towards a turbulent state, and we have also investigated the acceleration properties of a sheared flow where a current sheet is embedded, a model for the acceleration of the slow solar wind above helmet streamers.

Key words. Magnetohydrodynamics, Turbulence, Instabilities, Methods:numerical, Sun:corona, Sun:solar wind

1. Introduction

Magnetic reconnection plays a fundamental role in coronal heating, solar wind acceleration, and in the properties of turbulence and particle transport. Fast, inhomogeneous plasma flows are also ubiquitous throughout the heliosphere, arising both as a result of acceleration processes in the solar corona as well as from reconnection itself. The low to moderate values of plasma beta or ratio of plasma to magnetic pressure implies that flows resulting from reconnection will be supersonic. Moreover, the basic solar wind structure at solar minimum, consisting of high speed streams originating from the polar coronal hole, and a much slower stream surrounding the streamer belt above the solar magnetic equator, has a differential shear

which exceeds both the Alfvén and sound speed by factors of 5-10. The combined effects of resistive instabilities (Furth et al., 1963) and Kelvin-Helmholtz (KH) (Ferrari, 1998) type shear-flow instabilities may lead to much of the observed phenomenologies seen in the solar atmosphere and wind, from the spicules and macrospicules observed in the chromosphere and transition region, to coronal plumes and pressure-balanced structures in the solar wind (DeForest et al., 2001; McComas et al., 1995), to the acceleration of the slow solar wind above helmet streamers (Einaudi et al., 1999). Because the solar magnetic equator is tilted with respect to the solar rotation axis, the slow solar wind carries a current sheet which develops strong ballerina-skirt type undulations, associated also with the development of compressive structures due to the overlap of fast

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and slow streams, forming co-rotating interaction regions which develop shocks away from the sun. In the far region of the heliosphere, the current sheet must be carried across the heliospheric shock, recently crossed by the Voyager spacecraft (Krimigis et al., 2003), and the nature of the combined magnetic field and flow interaction there determines the structure of the heliosheath.

The problem posed by the numerical modeling of the interactions between high-mach number flows together with resistive instabilities, which may trigger the evolution of an ideally stable system, consists in the fact that the numerical techniques typically required to understand the two parts of the problem are at opposite extremes. To understand resistive instabilities and magnetic reconnection requires high order spectral-like techniques where the diffusion coefficients may be controlled explicitly, as the growth rate of resistive instabilities and reconnection rates scale explicitly as some power of the dissipative coefficients. In addition, magnetic reconnection in the coronal and heliospheric plasma is often controlled by kinetic effects rather than Spitzer resistivity, such as the Hall term, which introduce dispersion and add to the number of different wave-modes carried by the plasma (Birn et al., 2001). The wave-modes themselves may then become compressively unstable, with modulational and parametric effects. These terms are important as they decrease the dependence of the growth rate on the Reynolds numbers, but have not been used universally for coronal and heliospheric physics until today. On the other hand, the shocks which develop in high-mach number flows are best studied using numerical techniques explicitly designed to treat strong discontinuities, such as upwind methods with ENO (Essentially Non Oscillatory) interpolation polynomials.

In previous works, the choice most-often made was to allow numerical diffusion to explicitly dominate the resistive dynamic. However, in such cases current sheet structure and dynamics may be completely stabilised. An example in point is the acceleration of the slow solar wind above helmet streamers in the solar corona: an analysis of LASCO

coronagraph data (Wang et al., 1998) has revealed the presence of plasma density enhancements, called 'blobs', accelerating away from the Sun. These plasmoids are seen to originate just beyond the cusps of helmet streamers as radially elongated structures a few percent denser than the surrounding plasma sheet, and accelerate radially outward maintaining constant angular spans at a nearly constant acceleration up to the velocity typical slow wind velocity of 400km/s. In global standard global 3D MHD simulations, none of such intermittent features are observed. Instead, the closed helmet streamer is a dead zone for the wind, and the flow around it is completely laminar. While numerical simulations using spectral techniques of the wake region (Rappazzo et al., 2005) illustrate the resistive instability responsible for plasmoid formation, it is not possible to model the correct values of pressure and Mach numbers because of shock formation problems with this methods. We have also used spectral techniques to study the general question of resistive/KH instabilities in compressible magnetized plasmas with arbitrary geometries (Bettarini et al., 2006).

In the purpose of studying the dynamical activity following the destabilization of a current-sheet by magnetic diffusive processes in a compressible plasma, we have designed an Eulerian Conservative High Order Code (hereafter ECHO) which reconcile shock-capturing properties with high-order spectral-like differentiation schemes. The code integrate the conservative and compressible set of the MHD equations and is based on the Upwind Constrained Transport (UCT) methodology (Londrillo & Del Zanna, 2000; Londrillo & del Zanna, 2004) for treating the solenoidal condition for the magnetic field evolution. ECHO is able to handle the ideal and dissipative set of MHD equations as well as special and general relativistic ideal equations (Del Zanna et al., 2003, 2007). In the code we have introduced an explicit diffusivity of the magnetic field and we have implemented high-order implicit difference schemes (Lele, 1992) to approximate flux reconstruction and flux derivatives. The integration scheme on which ECHO is based allows to preserve the

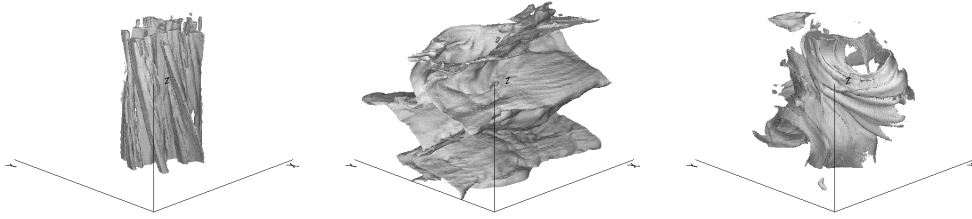


Fig. 1. Three dimensional representation of the plasma structure for three simulations of the tearing instability in a low plasma beta configuration in the late stage of the non linear evolution. The panels refer to three different initial configurations: A pressure equilibrium configuration with (left) and without (middle) guide field, and a force-free initial configuration (right).

local divergence-free condition for the magnetic field to within machine accuracy, to satisfy global conservation laws, to well resolve small-scale structures as well as to fully appreciate explicit resistivity against numerical diffusivity. Details on how the magnetic diffusivity has been handled and on the spectral-like properties of the designed numerical scheme can be founded in Landi et al. (2008).

2. Three-dimensional tearing instability

The basic dynamical evolution of a current-sheet is related to the so-called tearing instability whose linear properties has been first studied analytically by (Furth et al., 1963). Two-dimensional simulations have shown that the non linear evolution of the tearing modes produces turbulence by energy transfer toward higher wave-vector as well as an inverse cascade process that, because of the stretching of the most intense X-point formed during the linear regime, produces magnetic islands with larger and larger size (Hayashi, 1981; Malara et al., 1991). The transition to turbulence of a current-sheet can change drastically when moving from two to three dimensions: Many hydro and magnetohydro configurations can be subjected to the so-called secondary instability. The initial equilibrium configuration is driven by a two-dimensional instability (primary instability) toward a new configuration that is itself unstable to new three-dimensional ideal modes (secondary instability) that push

the system toward a turbulent state. In particular in a current-sheet the tearing modes can be overwhelmed by an ideal instability that has growth rates exceeding the reconnection rate (Dahlburg & Einaudi, 2002). The development of the secondary instability depends on the assumed initial configuration. As an example in a pressure equilibrium configuration of the current-sheet the presence of a guide-field, a constant magnetic field parallel to the current density, can have a stabilizing effect on the onset of the secondary instability (Onofri et al., 2004). Depending on the strength of the guide field the secondary instability can start or not (Dahlburg et al., 2005).

The scope of our work has been to extend the study of the linear and non linear evolution of the tearing instability to a compressible plasma both in the limit of high and low plasma beta (the ratio between the plasma and the magnetic pressure). Moreover we have extended the study to other possible initial configuration of the current-sheet, in particular by considering a force-free equilibrium which can be relevant in astrophysical applications such as the heliospheric current-sheet (Smith, 2001).

The numerical simulations has been performed by using the IBM SP Cluster 1600 at CINECA available through the INAF-CINECA agreement 2006-2007: High Performance Computing Resources for Astronomy and Astrophysics. An overview of the main results are shown in Fig. 1 and

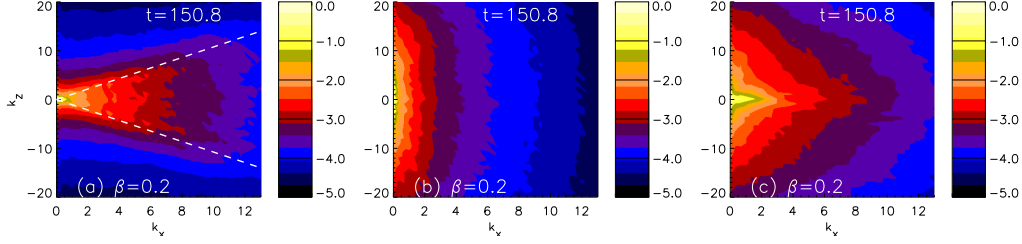


Fig. 2. Contour plot of the magnetic energy spectrum in logarithmic scale at a given instant during the non linear phase of three simulations of the tearing instability in the low beta regime starting from different initial equilibria: Pressure balance with guide field (a), pressure balance without guide field (b), and force-free equilibrium (c).

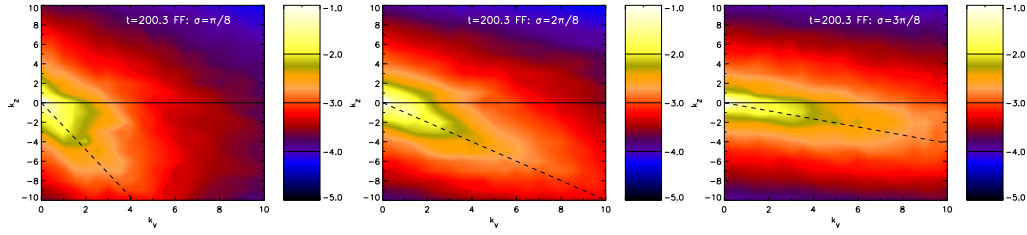


Fig. 3. Contour plot of the magnetic energy spectrum in logarithmic scale at a given instant during the non linear phase of three simulations of the wake-current sheet system evolution. All the simulations consider a force-free equilibrium as initial condition, but different angles between the wake and the magnetic field. The dashed lines drawn the angle between the z -axis and the asymptotic magnetic field for the three cases.

Fig. 2. Fig. 1 shows the plasma structure during the non linear phase of three different initial configurations: A pressure equilibrium configuration in presence of a guide field (left panel) and without guide field (middle panel), and a force-free configuration (right panel). All the simulations shown here refer to the case of a low beta plasma. It is clear from the figure that the fate of the late stage of the current-sheet evolution depend strongly on the initial equilibrium configuration. The pressure equilibrium configuration is subjected to the secondary instability which tends to destroy the basic state; the presence of a guide field have a stabilizing effects on the development of the secondary instability so that the coalescence process can be fully appreciated also in the three-dimensional case. The force-free configuration follows an intermediate path between the two previous cases: The system is destabilized by the secondary instability but,

the presence of a magnetic field component parallel to the current density vector inside the current sheet prevents the system to be completely dominated by the secondary instability. This different behaviours have their counterparts in the magnetic energy spectra evolution (not shown here, see Landi et al. (2008) for further details). In the non-linear regime the magnetic energy spectra, as shown in Fig. 2, are, in all three cases, anisotropic. When the coalescence process is dominating the system, as in the pressure equilibrium configuration with guide field, the magnetic energy spectrum is dominated by modes that obey to the resonant condition $\mathbf{k} \cdot \mathbf{B}_0 = 0$ (Malara et al., 1992; Onofri et al., 2004). In absence of the guide field, the secondary instability dominates the system and the magnetic energy spectrum reveals to have a preferential direction (here the z -direction) perpendicular to the magnetic field pressure gradient of the

initial configuration (the x -direction). The fact that in the force-free magnetic field equilibrium the secondary instability develops without overwhelming the primary resistive instability is reflected in the spectrum by the presence of a large dose of magnetic energy in both directions, parallel and perpendicular to the magnetic field gradient.

3. Three-dimensional current-vortex instability

In presence of both velocity shears and strong magnetic fields gradients a plasma system has a disposal a double reservoir of energy that provides the support for interactions between hydrodynamic and resistive instabilities. The incompressible two and three-dimensional instability of a current sheet embedded in a jet (or wake) flow, the jet(wake)-current sheet system, has been investigated intensively both linear and non linearly (Dahlburg & Karpen, 1994; Dahlburg et al., 1997, 1998; Dahlburg & Einaudi, 2001) with application to solar surges (Dahlburg & Karpen, 1994) and the acceleration of the slow solar wind (Einaudi et al., 1999). More recently compressive effects has been studied numerically in the linear regime (Dahlburg & Einaudi, 2000) and two-dimensional simulations have been performed in the context of the slow solar wind acceleration, plasmoid formation (Einaudi et al., 2001; Rappazzo et al., 2005) and more in general on the stability properties of the heliospheric current sheet (Bettarini et al., 2006).

With the facilities available through the INAF-CINECA agreement 2006-2007 we have performed three-dimensional simulations of a wake-current sheet configuration for a compressible plasma: as for the tearing instability we have considered different initial magnetic field and plasma configurations and we have consider also the effect of the angle between the velocity shear and the current sheet which generalize the stability properties of the heliospheric current sheet combined with the spiraling of the Sun's magnetic field (Bettarini et al., 2006). The use of numerical techniques which combine shock-capturing ca-

pabilities with high-order numerical resolutions has allowed us to use values for the sonic and Alfvén Mach numbers typical for heliospheric applications.

Several interesting features has been observed in the wake-current sheet evolution when passing from two to three dimensions. During the linear regime of the instabilities we observe a mode-selecting behaviour that depends on the angle of the asymptotic magnetic field with the velocity shear (Bettarini et al., 2008). This feature is also present in the fully developed non linear dynamic, as shown in Fig. 3 for three different simulations of the force-free equilibrium. Similarly to what observed for the tearing instability, for all the cases studied we observe formation of large scale structures that are either dominated by coalescence or by a superposition of kink-like instabilities (the secondary instabilities) according to the assumed equilibrium and/or to the magnetic field orientation (Bettarini et al., 2008). Finally, a general acceleration of the wake structure (the slow solar wind) is observed for all the equilibrium here considered, driven by tearing-like and Kelvin-Helmholtz like instabilities. An example of the wake profile evolution during in the non linear regime is given in Fig. 4.

4. Conclusions

We have designed an Eulerian Conservative High order code which integrate the conservative and compressible set of the MHD equations with the possibility to include dissipative effects like the magnetic diffusivity; the code employs high-order techniques both in space and time to obtain a good spectral resolution at high wave-number and to fully appreciate physical kinetic effects against numerical diffusivity. This code allows us to study the linear and non linear evolution of tearing and Kelvin-Helmholtz like instabilities in a compressible plasma which are thought to be an important ingredient in most phenomenologies observed throughout the heliosphere. In particular we have followed the three-dimensional evolution of the tearing instability and its transition towards a turbulent state and the evolu-

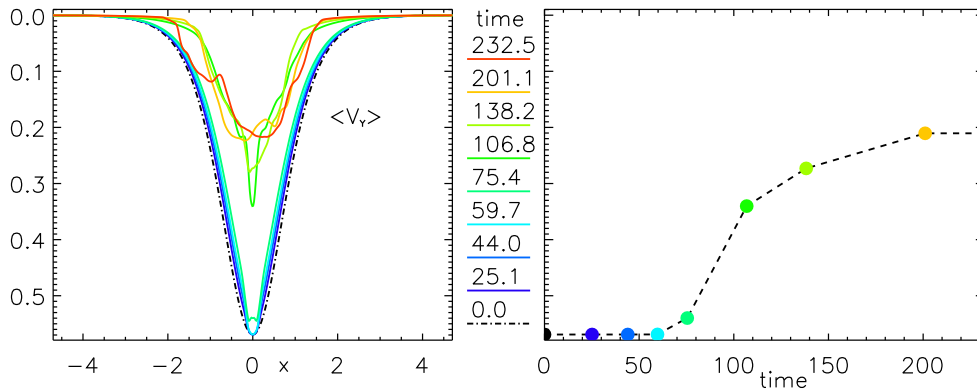


Fig. 4. On the left, time-evolution of the wake profile along the cross-stream direction once the other two directions has been averaged out. The mean wake velocity at the minimum as function of time is shown in the right panel.

tion of a current sheet embedded in a sheared flow, representative of the heliospheric current sheet embedded in the bimodal (fast and slow) solar wind. The general result has been the fact that the non-linear evolution of such systems depends strongly on the initial condition of the equilibrium configuration. In general an anisotropic magnetic energy spectrum is observed with energy carried by modes that have peculiar direction with respect both the current-sheet and the velocity shear. The spectra depend on the insurgence of secondary (ideal) instabilities or not as well as on the direction of the magnetic field with respect to the sheared flow. The non linear evolution of the wake-current sheet structure leads to the acceleration of flow which could be the origin of the slow solar wind.

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