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Low-energy collisions and dynamical evolution of asteroids

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Abstract. Low-energy collisions among Main Belt asteroids can play a role in the orbital evolution of these bodies. If an impact between a target and a projectile does not produce a disruption of the target, a transfer of linear momentum from the projectile to the target occurs, in addition to a transfer of angular momentum. While the angular momentum transfer produces a change of the target's rotational state, the linear momentum transfer produces an orbital change. An asteroid in the Main Belt is exposed to a steady bombardment by other Main Belt asteroids and meteoroids that are not able to destroy the target. Their cumulative effect is a continuous evolution of the target's orbit.
The physical and the mathematical foundation of the theory of the perturbations of the orbits by means of non-destructive collisions is summarized and the main results for Main Belt asteroids are described.

Key words. Minor planets, asteroids

1. Introduction

The most important process of physical evolution of Main Belt asteroids is due to the mutual collisions. The occurrence of mutual impacts among asteroids may explain several observed features of this population. The fact that asteroids undergo mutual collisions is based on several robust statistical analyses of the dynamical behavior of these objects (Wetherill (1967), Bottke & Greenberg (1993), Bottke et al. (1994), Dell'Oro & Paolicchi (1998), Vedder (1998)) and on many pieces of direct evidence, including the existence of dynamical families and the images of the heavily cratered surfaces of asteroids visited by space probes so far.

The orbits of Main Belt asteroids that are not characterized by a mean motion or secular resonance have semimajor axes, eccentricities and inclinations that oscillate around mean values, but the longitudes of the nodes and the arguments of perihelia, which describe the orientations of the orbits in space, vary more or less linearly with time. These effects are due to gravitational perturbations by the planets, mainly Jupiter. As a consequence, as the orbits of the asteroids continuously change their orientation in space, new mutual geometric configurations progressively take place, making it possible the production of mutual orbital intersections. More precisely, there is a non null probability that two different asteroid orbits may have, over an interval of time, a minimum distance less than the sum of the radii of the two objects. If the bodies transit around the

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mutual orbital nodes at the same time, a collision is possible. The average number of collisions per unit time that a Main Belt asteroid of radius R can have with a n other Main Belt asteroids is equal to nR^2P, where P is the so-called *mean intrinsic collision probability*, and its value is about 2×10^{-18} km^{-2}yr^{-1} (Bottke et al. 1994).

But what happens when two asteroids collide? The answer to this question engaged researcher for decades, since this is an extremely complicated problem. In general, one can expect that the outcome of a collision depends on the amount of energy delivered by the projectile. The energetic budget of a collisional event is usually described by the so-called *specific impact energy* of the projectile $Q = (1/2)(m/M)U^2$, where m is the projectile's mass, M the mass of the target and U the relative impact velocity. There is a general agreement that there exists an energy threshold Q^* for which, if $Q > Q^*$, the collision produces a catastrophic disruption of the target with the dispersion of its fragments. It is worth to recall that this is the process required for the formation of the so called asteroid families (Zappalà et al. 2002). Many models have been developed for the determination of the value of the Q^* threshold based either on laboratory experiments or theoretical considerations (Holsapple et al. 2002). The Q^* threshold depends on many parameters including target size, impact velocity, impactor incidence angle (impact parameter), target material, internal structure and and density.

For studies of the overall evolution of the Main Belt population from a statistical point of view, it is preferable to introduce for each value of the target's diameter D a mean value of Q^* obtained by averaging over possible values of the incidence angle, the impact velocity and a realistic density distribution. Of course, the function $Q^*(D)$ is a crucial ingredient for any study of the collisional evolution of the asteroids. In literature two extreme models can be found. At one end, representative of the so-called "hard" models, is the one developed by Benz & Asphaug (1999), based on numerical hydrocode simulations (Melosh et al. 1992). At the other end, representative of the "soft" models, is the one of Durda et al. (1998). In Fig. 1 the two models are represented, and it is clear how different they are. Up to two orders of magnitude can exist between the energy thresholds predicted by the two models. The models of Benz & Asphaug (1999) and Durda et al. (1998) represent two extreme scenarios among a variety of models available in the literature, and we refer to them in this paper just for this reason. We do not want to enter into a discussion about which one of the two is better. We use them only as terms of reference.

By producing the replacement of single asteroids by swarms of new ones consisting of their fragments, collisions with $Q > Q^*$ produce an evolution of the size distribution of the Main Belt population (Davis et al. 2002). Impacts below the energy threshold, on the other hand, are responsible of the change of other observable properties of the population. Apart from the role in forming binary asteroids and in producing impact craters on the surfaces, non-destructive collisions can have two different consequences. If we take into account the transfer of angular momentum, an impact produces a modification of the rotational state of the object, perturbing both the rotation period and the spin axis' orientation. This effect has been extensively investigated by different authors (Dobrovolskis & Burns (1984), Fujiwara (1987), Harris (1979), Harris (1990), Yanagisawa et al. (1996), Yanagisawa & Hasegawa (2000), Yanagisawa (2002)). On the other hand, the effect of the transfer of linear momentum has not been widely investigated so far. Only recently, a preliminary study about the effects of low-energy impacts on the orbital evolution of the objects has been performed (Dell'Oro & Cellino 2007).

2. Asteroid random walks

The basic idea of Dell'Oro & Cellino (2007) is that a single non-destructive collision between a target asteroid and a projectile body produces a transfer to the target of the linear momentum of the projectile. This, in turn, produces a corresponding modification of the target's orbit. More precisely, the variation of

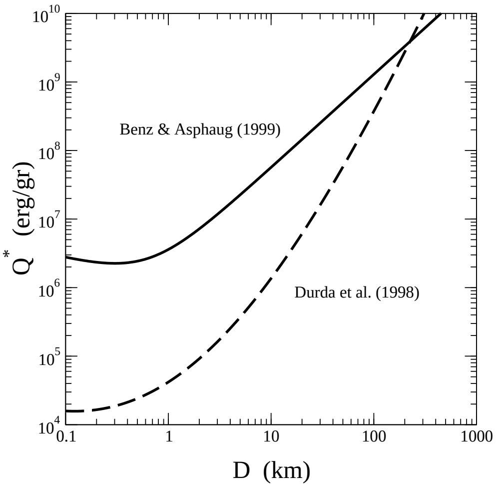

Fig. 1. Mean threshold Q^* versus target diameter for the models of Benz & Asphaug (1999) and Durda et al. (1998).

the orbital velocity of the target can be written down as $\delta \boldsymbol{v} = \eta(U)(m/M)\boldsymbol{U}$. The factor $\eta(U)$ is related to the physics of the impact. For a pure elastic head-on collision $\eta = 2$ ($m \ll M$), while in a pure inelastic collision $\eta = 1$. There are some indications that η in general can be greater than one when considering the ejection of the fragments in a crater formation (Ahrens & Harris 1994). Finally, the impulse $\delta \boldsymbol{v}$ produces a variation of the osculating elements semimajor axis (a), eccentricity (e) and inclination (I) of the orbit of the target that, since $\delta \boldsymbol{v}$ turns out to be much smaller than the orbital velocity of the target, can be computed with good accuracy using the well know Gauss' equations.

In the real world the target asteroid suffers much more than one non-destructive collision during its collisinal lifetime, *i.e.*, before being disrupted by an energetic collision. If δa_i is the variation in semimajor axis (the same can be repeated also for the eccentricity and inclination) due to a single collision, the total variation of a during an interval of time dt is d$a = \sum_i \delta a_i$, where the sum is extended to all collisions occurred during dt and due to projectiles in a given interval of mass dm. Since δa_i and the number N of collisions in a interval of time (that is the number N of terms in the previous sum) are random numbers, the total variation da is also a random number. The distribution of da depends on the distribution of the single impact variation δa, which in turn depends on the distribution of the impact velocity $\boldsymbol{U}$. For what concerns the distribution of the number N, we can assume to be Poissonian with mean $\langle N \rangle$, that can be written as $\langle N \rangle = R^2 P n \mathrm{d}t$ where P is the mean intrinsic probability of collision (Wetherill 1967), R the target's radius and n the number of asteroids in the interval of mass dm. The value of P and the distribution of $\boldsymbol{U}$ can be derived by means of a statistical theory of asteroid collisions and they have been well investigated in the past (Bottke et al. (1994), Dell'Oro & Paolicchi (1998)).

The crucial parameter is the number of projectiles in each interval of mass, that is the asteroid size distribution at small sizes. This is poorly known. The effect of gentle collisions on the orbit of km-sized asteroids depends on the population of objects smaller than one kilometer. One can assume that the size distribution of the asteroid below 5 km can be represented by means of a power law $\mathrm{d}N/\mathrm{d}D = D^{-\alpha}$, but different value of α are proposed in literature. Apart from the expected value for a collisionally relaxed population $\alpha = 3.5$ (Dohnanyi 1969), values of $\alpha > 3.5$ have been obtained by different authors on the basis of different direct and indirect evidences (Belton et al. (1992), Bottke et al. (2007), Tedesco & Désert (2002), Tedesco et al. (2005)). On the other hand, Ivezić et al. (2001) proposed a slope α around 2.3 for asteroids' sizes between 0.4 and 5 km. The situation is still uncertain, and for this reason Dell'Oro & Cellino (2007) tested different possible size distributions having slopes between 2.3 and 4.4. For exponents ≥ 4, the existence of a lower cut-off limit in the size distribution has to be assumed, for if the size distribution is extrapolated down to zero, the total corresponding mass of the Main Belt population would turn out to be infinite. Of course, this represents another important source of uncertainty, since there are not *a priori* indications of the size to which such new cut-off limit should correspond.

The evolution of the orbit of a particular asteroid due to the linear momentum transfer from a large number of non destructive collisions is a typical random walk path in the space of the orbital elements, similar to the Brownian motion of particles suspended in a fluid. For this reason the process in inherently stochastic so that the fate of a single object cannot be predicted in a deterministic way. In other words, it is not possible to predict which orbit a given object will have after some period of time, but it is possible to compute only a *distribution* of possible final orbits. In particular, we can compute which is the mean (expectation) $Mean(\mathrm{d}a)$ of the variation of the orbital element $\mathrm{d}a$ and the variance $Var(\mathrm{d}a)$ of the same variation. In general, we can image that the set of possible final orbits is a cloud of possible values of a centered around $a_0 + Mean(\mathrm{d}a)$, where a_0 is the initial value of the semimajor axis, and having a size of the order of $Stdev(\mathrm{d}a) = Var^{1/2}(\mathrm{d}a)$, that is the standard deviation of $\mathrm{d}a$.

Two remarks are necessary. First, it turns out in general that $Mean(\mathrm{d}a) \neq 0$, that is the object suffers a systematic shift of its orbital element, that can be either positive or negative. Second, the variation in time of $Mean(\mathrm{d}a)$ and $Var(\mathrm{d}a)$ has the properties of a typical random walk, that is both $Mean(\mathrm{d}a)$ and $Var(\mathrm{d}a)$ are proportional to $\mathrm{d}t$, so the size of the cloud of the final orbits is $Stdev(\mathrm{d}a) \propto (\mathrm{d}t)^{1/2}$. In Dell'Oro & Cellino (2007) many numerical simulations have been done in order to compute the values of $Mean(\mathrm{d}a)$ and $Stdev(\mathrm{d}a)$ after a given interval of time. The main results are the following:

- The values of $Mean(\mathrm{d}a/a)$ and $Stdev(\mathrm{d}a/a)$ are affected much more by the slope α of the asteroid size distribution than by the break-up physics, although in general, as intuitive, the value $Mean(\mathrm{d}a/a)$ and $Stdev(\mathrm{d}a/a)$ for the physics of Benz & Asphaug (1999) can be one orders of magnitude larger than assuming the physics of Durda et al. (1998), because in the former case the objects are more resistant to destruction and so they have more time for their orbits to evolve.
- For a given value of the slope α of the size distribution, the values of $Mean(\mathrm{d}a/a)$ and $Stdev(\mathrm{d}a/a)$ are strongly dependent on the target's size. The complex interplay between the size distribution of the projectiles, the probability of collision proportional to R^2 and the dependency between Q^* and the target size make $Mean(\mathrm{d}a/a)$ and $Stdev(\mathrm{d}a/a)$ in some cases increasing with D and in some cases decreasing with D (Fig. 2).
- The values of $Mean(\mathrm{d}a/a)$ and $Stdev(\mathrm{d}a/a)$ for a given target size D are strongly dependent on the assumed size distribution. Different values of α imply many orders of magnitude of difference in the rates of variation. On the other hand, the values of $Stdev(\mathrm{d}a/a)$ turn out to be only poorly dependent on α for targets larger than about 30 km.
- The values of $Mean(\mathrm{d}a/a)$ for $\alpha > 3.5$ are comparable with the mean drift rate induced by the Yarkovsky effect (Bottke et al. 2002), and they can be even larger than the Yarkovsky drift for very steep size distributions ($\alpha > 4$). For asteroids larger than 30-40 km the value of $Stdev(\mathrm{d}a/a)$ after 1 Myr of evolution is larger than the Yarkovsky drift, regardless the assumed size distribution (α).
- Unlike the Yarkovsky effect, the collisionaly-driven orbital mobility of the asteroids affects, more or less to the same extent, also the eccentricities and the inclinations and not only the semimajor axis (Fig. 2). Due to the orbital distribution of the Main Belt asteroids, the relative velocity of impact turns out to have the tangential and normal to orbit components on the average larger than the component along the radial direction. In this case, low-energy collisions should produce an orbital variation that should affect more eccentricities and inclinations than semimajor axes. On the other hand, in lateral impacts, one can assume that the linear momentum transferred to the target is due mainly to the normal component

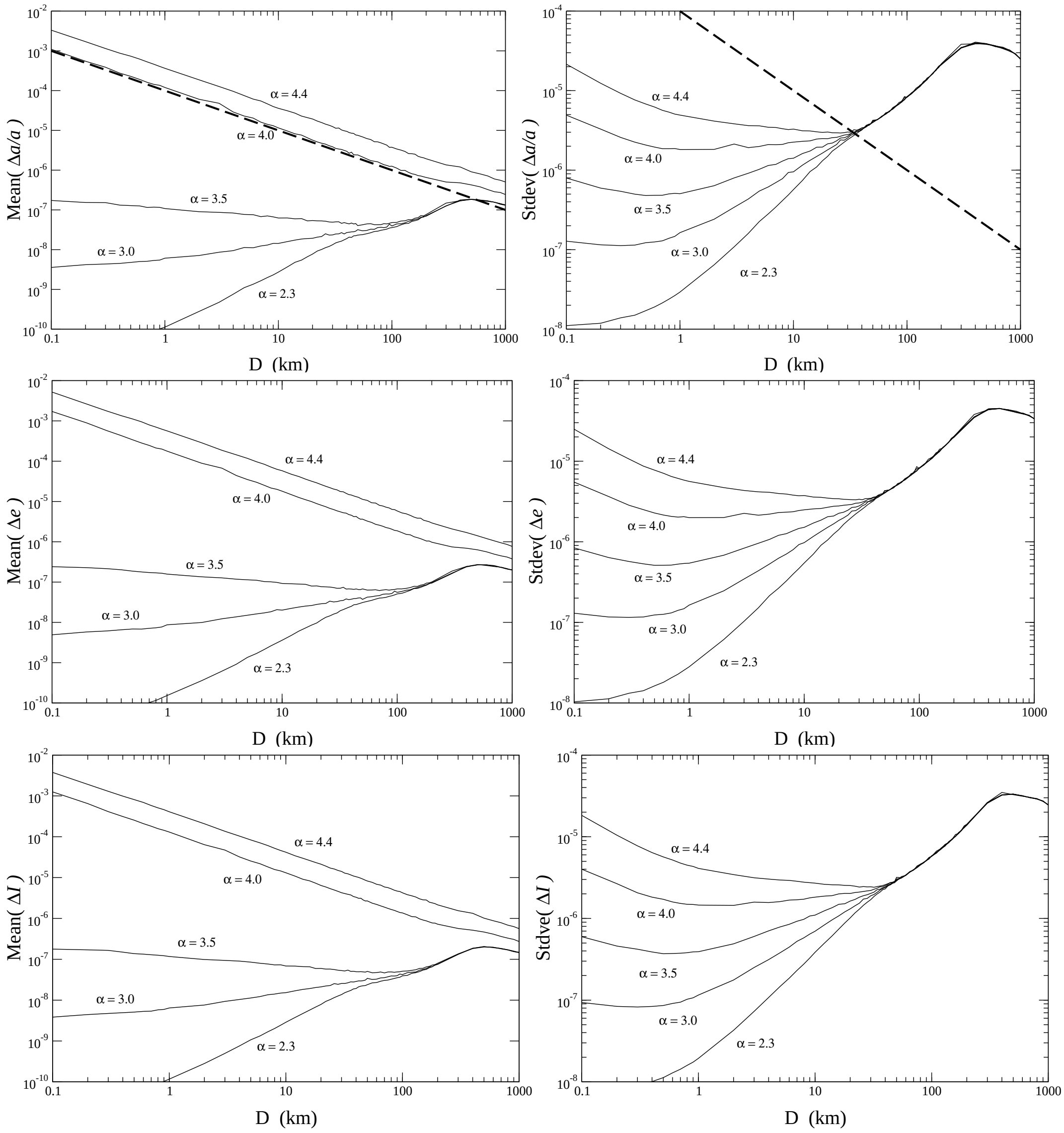

Fig. 2. Mean values (left) and standard deviations (right) of the variation of the orbital elements as a function of target's diameter (in km), for Main Belt asteroids. All plots describe a time evolution of 1 Myr. The variation of $\Delta a/a$ (top), Δe (center) and ΔI (bottom) are shown. The values of α are the different slopes of the assumed incremental size distributions of the asteroids. These plots have been produced by assuming the break-up physics of Benz & Asphaug (1999).

$\boldsymbol{U}_{\perp} = (\boldsymbol{n} \cdot \boldsymbol{U})\mathbf{n}$ of the impact velocity $\boldsymbol{U}$ with respect to the target surface ($\boldsymbol{n}$ is the unit vector normal to the surface at the point of impact). It is then possible to show that the distribution of $\boldsymbol{U}_{\perp}$ is more or less isotropic, producing changes of the three orbital element more or less in the same way.

3. Conclusions

The orbital mobility of asteroids due to mutual non-destructive collisions can represent, in addition to the Yarkovsky effect and the perturbations due to mutual gravitational interactions (Carruba et al. 2003), another possible mechanism of orbital evolution, different from the gravitational perturbations of the planets. These mechanisms seem to play a crucial role for the evolution of the asteroids.

In particular, the Yarkovsky effect is currently thought to be the major responsible for the delivery of material from the asteroid Main Belt to the region of the Near Earth Asteroids. The Yarkovsky force, producing a drift in semimajor axis, pulls Main Belt asteroids into unstable zones of chaotic orbital motion caused by mean-motion and secular resonances with the major planets. Asteroids in these conditions are quickly removed from the Main Belt and may be led towards the Inner Solar System, in such a way as to produce a steady supply to the population of Near Earth Asteroids (Bottke et al. 2002). Based on this scenario, models of formation of Near Earth Asteroids have been developed (Morbidelli & Vokrouhlický 2003). Moreover, it has been pointed out that the Yarkovsky effect can play a very important role for the evolution of asteroid families. In the past, Zappalà et al. (1996) had explored the possibility to reconstruct the original fragments' ejection velocities in family-forming break-up events, on the basis of currently observed proper orbital elements of family members. But in a Yarkovsky scenario, the currently observed distributions of orbital elements have been mostly dominated by an orbital post-formation, Yarkovsky-driven evolution. While the semimajor axes are directly affected by the Yarkovsky effect, the eccentricities and inclinations should be progressively modified by a resonance-crossing taking place due to the Yarkovsky-driven drift in semimajor axis. Modeling asteroid families by including in the computations the Yarkovsky effect makes it possible a natural interpretation of some observed features of observed family configurations, and leads also to derive a chronology of the families themselves (Bottke et al. (2001), Vokrouhlický et al. (2006)).

On the other hand, some problems remain unsolved. The efficiency of the Yarkovsky effect has been recently scaled down, due to a complex interplay with other effects involving the evolution of the spin states due to thermal forces (the so called YORP effect) and by the dynamical spin-orbit interaction (Vokrouhlický et al. 2006). Another problem is that, although the Yarkovsky effect provides a natural interpretation of the observed distribution of semimajor axes of asteroid family members, so far no conclusive demonstration has been provided that a pure Yarkovsky scenario can fully explain the observed distributions of the eccentricities and inclinations (Cellino et al. 2004). As we have seen before, the orbital mobility induced by non destructive collisions affects all the three orbital elements in the same way and it can explain the observed spreading of the eccentricities and inclinations of family members, in the same way as it describes a progressive spreading of the semimajor axes. New numerical computations are required in order to verify this hypothesis, in order to assess whether, together with thermal and dynamical mechanisms, low-energy collisions among asteroids may be the "missing ingredient" to build a comprehensive theoretical framework to model all the major observed properties of asteroid dynamical families, and to develop a new refined model of the overall collisional evolution of Main Belt asteroids.

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